**Radiative processes in Astrophysics-** F. Perrotta, SISSA. 2023-2024

**Problem set #2.**

# Bremsstrahlung radiation.

**Problem 11.** For a single electron-ion interaction, and for a fixed electron velocity, the maximum total radiated energy corresponds to the minimum allowed impact parameter.

Show that this cannot exceed the kinetic energy of the electron, either for $b_{min} = b_{min,1}$ and for $b_{min} = b_{min,2}$ (useful to remember that $2\pi\, e^2 / 2m_e c = \alpha = 1/137$)

**Problem 12** . Consider an electron moving at a speed of v =1000 km $s^{-1.}$

a) Calculate the frequency of the emitted radiation in case it would radiate all its kinetic energy in a single interaction (a single photon).

b) In case this electron belongs to a population of particles at thermal equilibrium for which the typical speed is v = 1000 km $s^{-1}$, determine which is the temperature of the plasma.

**Problem. 13.** Evaluate $b_{min}$ for the warm plasma in the Lagoon nebula observed at 1 GHz, selecting if we have to use the classical or quantum value.

**Problem 14**. An HII region is generated by an O-type star. Suppose it has a spherical volume of radius 1pc filled by pure Hydrogen plasma, at a temperature T= $1.6 \times 10^4$ K and numerical density $n_e$=10 $cm^{-3}$. Calculate the specific emissivity and the bolometric one. If the star was removed, after how much time will the region stop to emit? If the region is at a distance 250 pc, calculate the flux observed by a radiotelescope at 10 GHz. .(Assume the emission is optically thin at this frequency).

Useful formulae: flux $S_\nu = \frac{\varepsilon_\nu(T)\cdot V}{4\pi D^2}$ $\qquad$ $L = \varepsilon(T) \cdot V$

**Problem 15. (Worked example)** Suppose we take the matter in our own Galaxy and heat it to $T=10^6$ K in a spherical region of order the Sun's distance from the Galactic center, R≈ 10 Kpc. Assume the cloud to be **fully ionized H**, with uniform density and total mass $M=10^{11} M_{\odot}$.

What electron density is required to make the cloud optically thick?

**Problem 16.** The mean density of free electrons in the Universe is ~ $10^{-5}$ $cm^{-3}$. Show the Universe is optically thin to electron scattering. UV photons from a distant cloud like the one above would be nevertheless very hard to see. Why?

**Problem 17** Derive a general expression for the cooling time for a thin plasma, via thermal bremsstrahlung, in terms of the electron density $n_e$ , the ion density $n_i$ , ion charge Z, radius R and temperature T.

# Synchrotron radiation .

**Problem 18:** Indicating with β = v/c , show that:

the acceleration is given by $a = \frac{q\beta B}{\gamma m}$ ;

the radius of gyration is $r = \frac{\gamma m c^2 \beta}{qB}$ ;

the period of gyration is $\mathrm{P} = \frac{2\pi\gamma m c}{eB}$ ;

the frequency of gyration is $\varpi_B = \frac{2\pi}{P} = \frac{qB}{\gamma m c}$

**Problem 19.** Show that the radiation emitted by a relativistic electron moving in a magnetic field is $P = \frac{2}{3} r_0^2\, c\, \beta_\perp^2 \gamma^2 B^2$ , with $\beta_\perp = \frac{v_\perp}{c} = \beta \sin\alpha$ , and $r_0 = e^2/m_e c^2$ (≈ 2.8 x $10^{-15}$ m) is the classical electron radius .

**Problem 20** A relativistic charge in its own reference frame is emitting cyclotron power with a dipolar distribution $P(\theta) = P(0) \cos^2\theta$. Show that at 99.9% of the speed of light :

- Radiation originally emitted at $45^{\circ}$ to the direction of the velocity is seen by an external observer only $1^{\circ}$ away from it.
- The points of null radiation, at $90^{\circ}$ to the velocity in the electron's frame, appear directed $2.5^{\circ}$ away from it to the external observer.
- Show that the null power points of the emission in the electron's frame in general appear at an angle $\gamma^{-1}$ radians in the external frame.

**Problem 21:** The nebula emits X-rays at $10^{5}$ eV ($\sim 10^{19}$ Hz) , which can be associated with the peak of the synchrotron spectrum at 0.29 $\nu_c$.

The magnetic field in the nebula has a strength of $B \approx 10^{-4}$ Gauss.

Compute the energy of the electrons producing this radiation, and the synchrotron power radiated per electron if they are isotropic. How long can the electrons radiate X-rays at this rate? Show that this is much shorter than the known age of the Crab (900 years).

**Problem 22:** The synchrotron spectrum of the Crab shows a **change of slope** at around $10^{15}$ Hz ($\sim$ 4 eV), thought to be due to the high energy electrons, which radiate their energy away in a time shorter than the age of the nebula. Show that at this frequency the electrons have a lifetime which is *about* the same as the age of the Nebula, if the magnetic field strength is $B = 10^{-4}$ Gauss.