Radiation Scattering processes

SISSA

- Thomson scattering
- Compton scattering
- Klein-Nishina cross section
- Inverse Compton: spectrum, multiple scattering, energy losses, cooling time
- Synchrotron-Self Compton
- Compton catastrophe
- Comptonization.

Reference books :

Nobili, L., 2002, *Processi Radiativi ed Equazione del Trasporto nell'Astrofisica delle alte energie,* ed. Cleup, Padova
Longair, M.S., 1992, *High Energy Astrophysics*, Cambridge University Press
Ghisellini G., 2013, *Radiative Processes in High Energy Astrophysics,* Springer ed.

Useful units and formulas in CGS (Gaussian units)

Electric charge is measured in statC (or «esu»):

1 statC = 1 $g^{1/2}$ $cm^{3/2}$ s^{-1}

B(magnetic field) in Gauss, with 1 G = 1 $g^{½}$ $cm^{-1/2}$ s^{-1} ; 1 Tesla= 10^4 G

1 erg = 1 g cm^{2} s^{-2}

1 eV= 1,60218 × 10^{-12} erg = 1,60218 10^{-19} J ; 1 eV= 2.418×10^{14} Hz (from E=hν)

$r_0 = e^2/m_e c^2$ (≈ 2.8 × 10^{-13} cm);

$\sigma_T = 8\pi r_0^2/3 = 6.6524 \times 10^{-25}\ cm^2$;

e = electron charge = 4.8032 ×10^{-10} statC

m_e= 9.1093937 ×10^{-28} g;

h=4.13 ×10^{-15} eV s = 6.26 ×10^{-27} erg s

c ≈ 2.998 × 10^{10} cm s^{-1}

h ≈ 6.626 ×10^{-27} erg s

Boltzmann constant $k \approx 1.381 \times 10^{-16}$ erg K^{-1} = 8.61 × 10^{-5} eV K^{-1}

$m_e c^2$ ≈ 34.3 erg

Photon-electron scattering: overview

The simplest interaction between photons and free electrons is scattering. When the energy of the incoming photon (as seen in the **frame of the electron**) is small w.r.t. the electron rest-mass energy, $h\nu_i \ll m_e c^2$ (0.511 Mev) the process is called **Thomson scattering**, which can be described in classical electrodynamics, i.e. radiation can be seen as a wave. Thomson scattering is elastic, since the electron acts as a passive diffusor, without taking any energy from the incident photon.

When the electron is at rest (or non relativistic) and photon energy is $h\nu_i \geq m_e c^2$ (electron rest energy), quantum effects have to be taken into account. Quantum effects appears through the kinematics of the scattering process, which occur because a photon possesses a momentum as well as an energy . The momentum exchange between photon and electron manifest as a recoil of the electron , which *subtracts* energy to the incident photon (Klein-Nishina regime), making the scattering no longer elastic. This is the **direct Compton (or just Compton) scattering**, and it is a heating process for the electrons. The recoil is responsible of the electrons heating, and it represents the energy lost by the photons.

When the electron is relativistic, the momentum exchange in the Compton scattering has to be treated relativistically. In this case, IF the electron energy is greater than the photon energy, the typical photon gains energy, cooling the electron gas, even if there are some arrangements of angles for which it looses part of its energy. This is the **inverse Compton scattering.**

Thomson scattering

Thomson scattering is elastic diffusion of electromagnetic radiation by a free, nonrelativistic charge.

Charge acceleration due to the $\vec{E}$ of the incident wave (assumed to be linearly polarized). The acceleration is in the direction of the oscillating electric field, resulting in electromagnetic dipole radiation. Most radiation in a direction perpendicular to charge motion. This radiation is polarized on the plane containing the charge acceleration (i.e., the incident polarization) and the observing (scattering) direction. Be $\theta = \widehat{\mathbf{d}\,\mathbf{k_f}}$ (*not* the scattering angle Θ ! $\Theta = \frac{\pi}{2} - \theta.$)

The force on the charge by an electromagnetic wave is :

$\mathbf{F} = \mathrm{q}\,\hat{\boldsymbol{\varepsilon}}\,\mathrm{E}\sin\omega\,\mathrm{t}$, the dipole moment $\mathbf{d} = \mathrm{q}\mathbf{r} \rightarrow \ddot{\mathbf{d}} = \mathrm{q}\ddot{\mathbf{r}} = \frac{q^2}{m}\hat{\boldsymbol{\varepsilon}}\,\mathrm{E}\sin\omega\,\mathrm{t}$

Using the Larmor's formula, the time averaged power scattered into dΩ=senΘdΘdφ is:

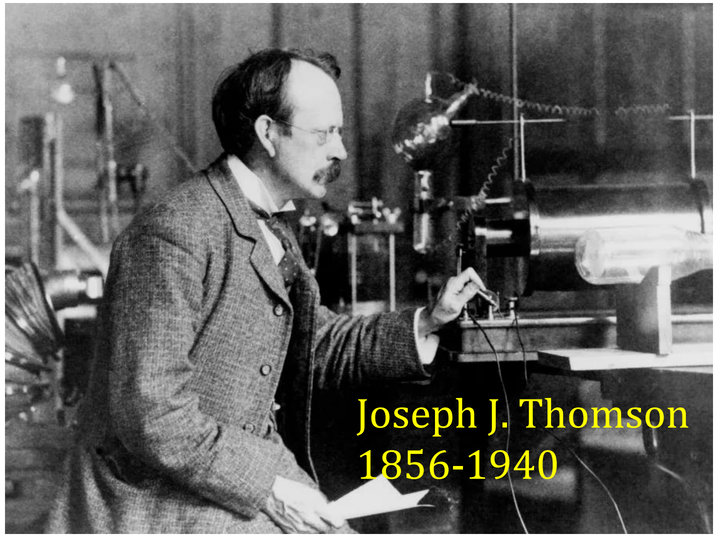
$\left\langle\frac{dP}{d\Omega}\right\rangle = \langle S\rangle \frac{d\sigma}{d\Omega}$ (**S** being the Poyinting vector) . For electrons,

$\frac{d\sigma}{d\Omega} = \frac{e^4}{m_e^2 c^4}\sin^2\theta = r_0^2\sin^2\theta$ (max in the front and back directions)

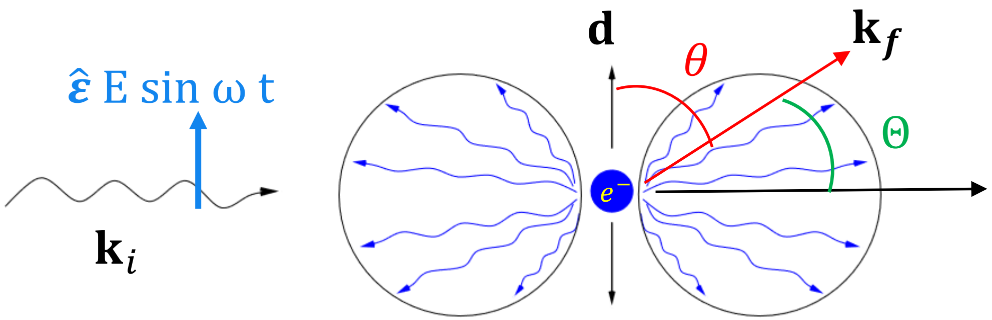
Upon integration over the solid angle, $\sigma_T = \frac{8\pi}{3}r_0^2 = 6.65 \times 10^{-25}\mathrm{cm}^2$

$r_0 = 2.82 \times 10^{-13}$cm (classical electron radius).

$$\frac{dE}{dt} = c\sigma_T\,\mathrm{U}_r$$



Nobel prize in Physics, 1906: Discovery of sub-atomic nature of electrons.



Electron oscillates sinusoidally.
Azimuthal symmetry around **d**

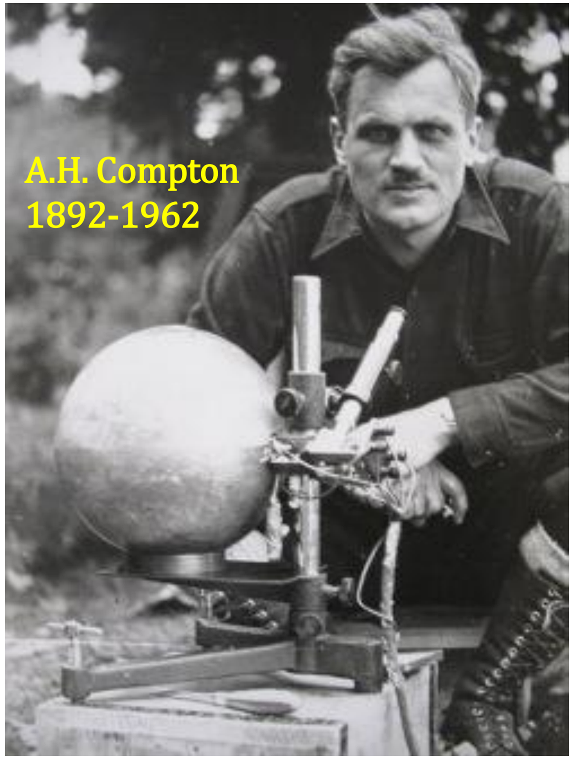
Going beyond the Thomson limits: Compton scattering

It was originally experimentally demonstrated by Arthur H. Compton that the wavelength of X-rays increases when they are scattered against electrons. Classical wave description of light scattering (Thomson scattering) cannot explain any shift in frequency. Light must behave as **particles** in order to explain the Compton scattering. This effect is due to the quantum-mechanics radiation properties, since photons carry momentum as well as energy : Compton Scattering is a scattering phenomenon between the photon and a charged particle that causes momentum exchange between the photon and the electron. The momentum exchange causes a recoil of the target charge (electron), which subtracts energy from the incident photon, redshifting its frequency.

The Compton effect is the decrease in energy (increase in λ) of a high energy (X – or γ – ray) photon, as it interacts with a free electron. The quantum properties of radiation become non negligible when $h\nu_i \gtrsim m_{charge}c^2$.

It is one of the main interaction channels between radiation and hot, rarefied plasmas. Arthur H. Compton was awarded the Nobel Prize in Physics in 1927.

Although nuclear Compton scattering also exists, what is meant by Compton scattering usually is the interaction involving *free electrons only*.



Compton scattering

- **Particle Quadrimomentum** : $p^{\mu} = (p^0, \mathbf{p}) = (\gamma mc, \gamma m\mathbf{v})$ with Lorentz factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = (1-\beta^2)^{-1/2}$, being m the particle's rest mass and $\beta \equiv \frac{v}{c}$.

The quantity $p_{\mu}p^{\mu} = (p^0)^2 - \mathbf{p}\cdot\mathbf{p} = (\gamma mc)^2 - (\gamma m v)^2 = \gamma^2 m^2 c^2 (1-\beta^2) = \gamma^2 m^2 c^2 \cdot \gamma^{-2} = m^2 c^2 =$ relativistic invariant

- **Particle energy** : $\mathcal{E} = c\, p^0 = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$ [Notice, mc^2 is the *rest energy*, due to the particle having a non-zero mass].

Since $p^0 = \sqrt{m^2c^2 + \mathbf{p}\cdot\mathbf{p}}$, the relativistic energy of a particle moving with velocity $\mathbf{v}$ is :

$$\mathcal{E} = c\, p^0 = c\sqrt{m^2c^2 + \gamma^2 m^2 v^2} = \sqrt{m^2c^4 + p^2c^2} = mc^2\sqrt{1+\gamma^2\beta^2}$$ ($p^2 \equiv \mathbf{p}\cdot\mathbf{p}$, *spatial components of momentum*)

Kinetic term

- **Photon Quadrimomentum:**

$p^{\mu}_{photon} = \left(\frac{h\nu}{c}, \frac{h\nu}{c}\boldsymbol{n}\right)$, where $\boldsymbol{n}$ is the unit vector corresponding to the propagation direction (i.e., the direction of the spatial wavevector $\mathbf{k} = \frac{2\pi\nu}{c}\boldsymbol{n}$. The four vector $k^{\mu} \equiv \left(\frac{2\pi\nu}{c}, \mathbf{k}\right) = \frac{2\pi}{h} p^{\mu}_{photon}$, is a *null* vector , $k^{\mu}k_{\mu} = 0 = \left(p^{\mu}_{photon} p_{\mu,photon}\right)$

- **Photon energy**: $\mathcal{E}_{photon} = c p^0_{photon} = h\nu$

Compton scattering

We can then consider the direct interaction between an electron and a single photon, assuming that they will exchange energy in such a way to conserve the total four-momentum (i and f labelling initial and final values):

$$p^{\mu}_{photon,i} + p^{\mu}_{electron,i} = p^{\mu}_{photon,f} + p^{\mu}_{electron,f}$$

When the target electron is initially at rest, the 4-momentum conservation gives:

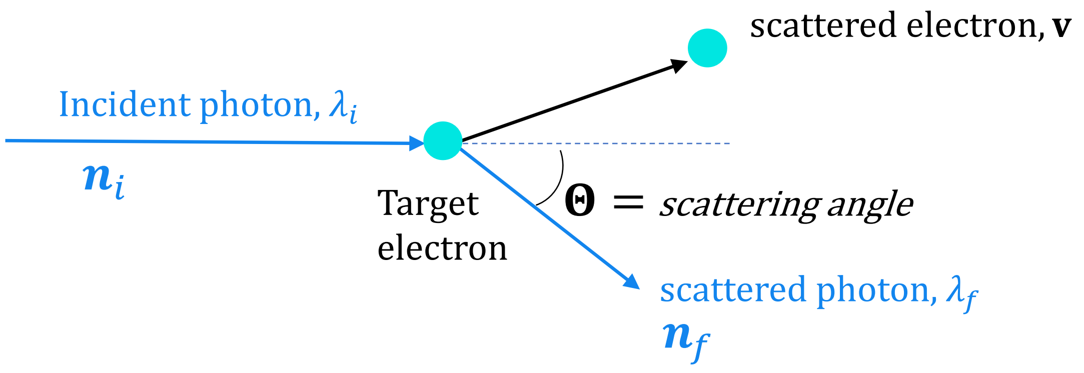
$$\begin{cases} h\nu_i + m_e c^2 = h\nu_f + \sqrt{m_e^2 c^4 + p^2_{electron,f}\, c^2} & \text{(energy conservation)} \\ \frac{h\nu_i}{c}\,\boldsymbol{n}_i = \frac{h\nu_f}{c}\,\boldsymbol{n}_f + \mathbf{p}_{electron,f} & \text{(spatial momentum conservation)} \end{cases}$$

Being $\boldsymbol{n}_i \cdot \boldsymbol{n}_f = \cos\boldsymbol{\Theta}$, one writes an equation for $p^2_{electron,f}$. Than take the square of the energy conservation equation. With a little algebra, we obtain:

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1 - \cos\boldsymbol{\Theta})$$

$\lambda_C = \dfrac{h}{m_e c} = 0.02426$ Å Compton wavelength for electrons

$$\lambda_f - \lambda_i = \lambda_C(1 - \cos\boldsymbol{\Theta})$$



Compton scattering

The energy of the scattered photon is $h\nu_f = \frac{h\nu_i}{1+h\nu_i(1-\cos\Theta)/m_e c^2}$, function of initial energy photon and scattering angle.

Defining the unitless photon energies as $\epsilon \equiv \frac{h\nu}{m_e c^2}$,

$$\epsilon_f = \frac{\epsilon_i}{1+\epsilon_i(1-\cos\Theta)}$$

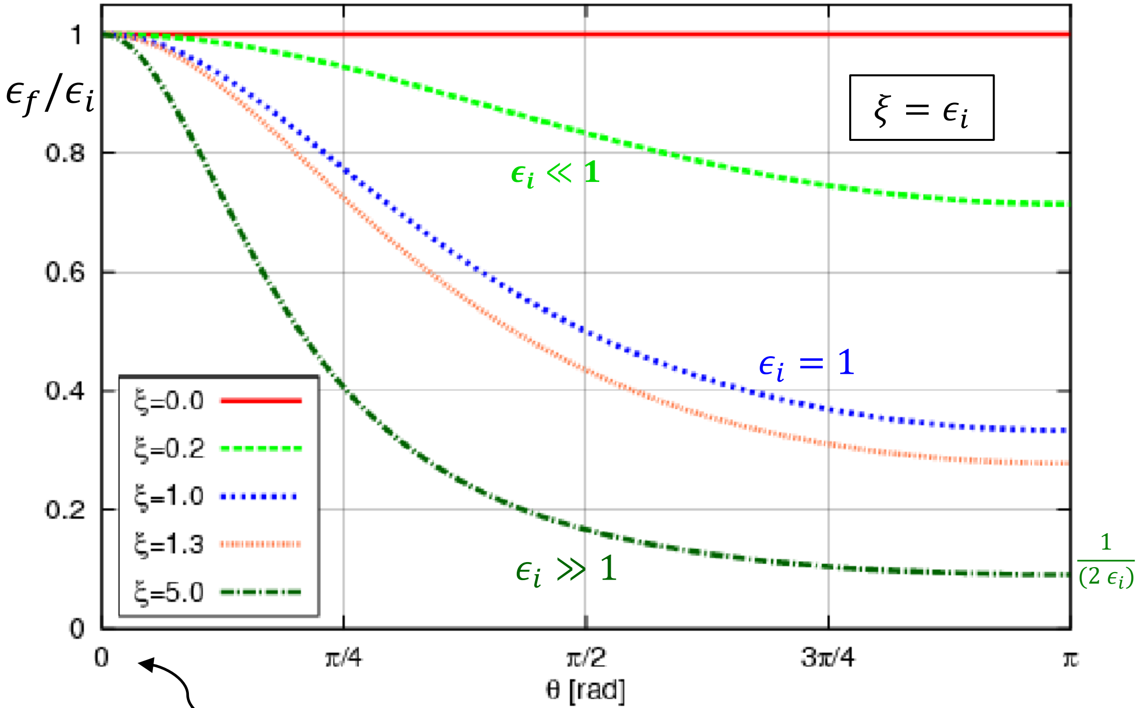
Energy loss ↔ scattering angle

Note that:

- The photon always looses energy, unless $\mathbf{\Theta} = 0$ (in which case $\epsilon_f = \epsilon_i$). The photon energy shift arises from the recoil of the electron initially at rest, and becomes significant only when $\boldsymbol{\epsilon_i} \gtrsim \mathbf{1}$: quantum mechanical effect. $E_{recoil} = (\epsilon_f - \epsilon_i)\, m_e c^2$. It is a heating mechanism.

- For $\boldsymbol{\epsilon_i} \gg \mathbf{1}$ $(\lambda_i \ll \lambda_C)$ and $\cos\mathbf{\Theta} \neq 1$, $\epsilon_f \to (1-\cos\mathbf{\Theta})^{-1}$. In this case, the scattered photons carries information about the scattering angle, rather than about the initial energy. E.g., for $\mathbf{\Theta} = \pi$ and $\epsilon_i \gg 1$, the final energy is $\epsilon_f = ½$, which is $h\nu_f = 1/2\, m_e c^2 = 255$ keV.

- The scattering is *close* to elastic when $\boldsymbol{\epsilon_i} \ll \mathbf{1}$ $(\epsilon_f \sim \epsilon_i)$, i.e. $h\nu \ll m_e c^2$ $(\lambda_i \gg \lambda_C,)$ (Thomson limit). Tiny electron recoil, ~ elastic scattering.

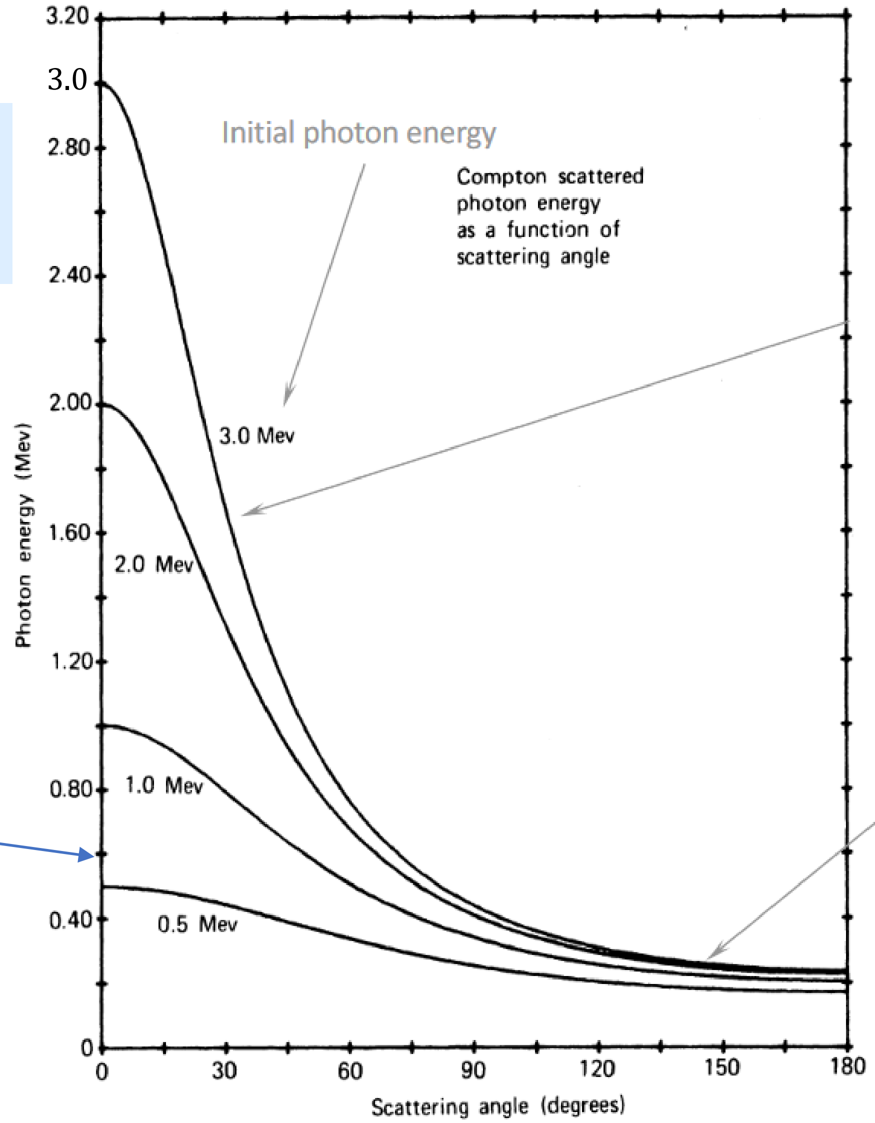


$\mathbf{\Theta} = 0$, forward elastic scattering

Compton scattering- Energy transfer

Example. Energy of the scattered photon vs scattering angle, for different initial photon energies.

$h\nu_i = 0{,}5\ \text{MeV} = m_e c^2$
(Hard X – Gamma)



Small angle scattering:

Energy carried by the scattered gamma ray depends strongly on scattering angle

Large angle scattering:

Energy carried by the scattered gamma ray depends only weakly on scattering angle

Figure from Page 321, Radiation Detection and Measurements, Third Edition, G. F. Knoll.

Compton scattering- Energy transfer

The **maximum recoil energy** is obtained setting $\theta = \pi$ (full backscatter), when ϵ_f is minimum, thus maximizing the energy transferred to the electron :

$$E_{max} = \frac{2h\nu_i}{2+\frac{m_e c^2}{h\nu_i}}$$

This maximum energy is called «Compton edge».

The Compton edge is a feature of the spectrograph that results in a scintillator (or in a detector), from the Compton scattering of photons with a scattering angle of 180° , which escape the detector. When a gamma ray scatters off the detector and escapes, only a fraction of its initial energy can be deposited in the sensitive layer of the detector. The scattering angle determines how much energy will be deposited in the detector. This leads to a spectrum of energies. The Compton edge energy corresponds to full backscattered photon.

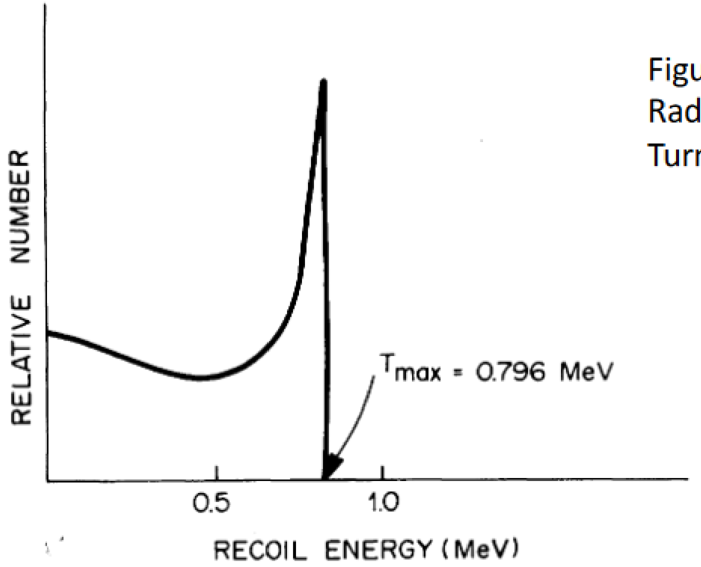


FIGURE 8.5. Relative number of Compton recoil electrons as a function of their en for 1-MeV photons.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p180.

Compton scattering- The Klein-Nishina cross section

When diffusion involves a large number of electrons and/or photons, the problem has to be treated statistically, studying the effects of multiple scatterings weighted by their cross section. Polarization of the incoming radiation makes the scattered radiation no longer isotropic with respect to the azimuthal angle φ. The differential cross section is in the ERF and it is given by the Klein-Nishina formula, averaged over φ:

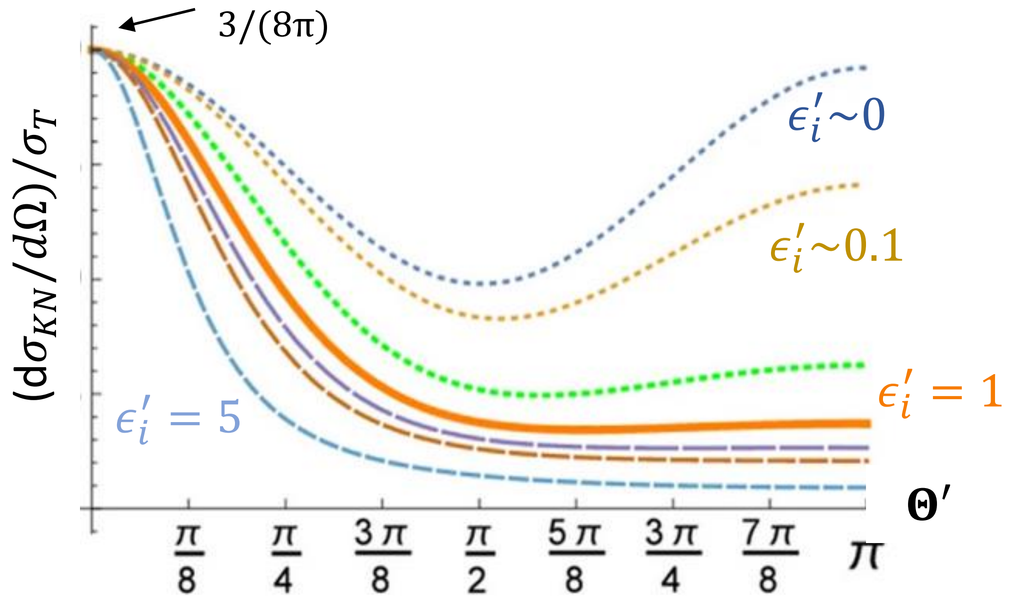
$$\frac{d^2\sigma_{KN}}{d\epsilon'_f d\Omega'_f} = \frac{3}{16\,\pi}\sigma_T \left(\frac{\epsilon'_f}{\epsilon'_i}\right)^2 \left(\frac{\epsilon'_i}{\epsilon'_f} + \frac{\epsilon'_f}{\epsilon'_i} - \sin^2\boldsymbol{\Theta}'\right)\delta(\epsilon'_f - \epsilon_*) \quad , \quad \cos\boldsymbol{\Theta}' = \mathbf{k}'_i \cdot \mathbf{k}'_f, \quad \text{with } \sigma_T = \frac{8}{3}\pi r_0^2 = 6.65\times 10^{-25}\text{cm}^2 \text{ (Thomson)}$$

Here ϵ'_f is not arbitrary , constrained by the energy conservation to ϵ'_i and $\mathbf{k}'_i$: $\epsilon_* = \frac{\epsilon'_i}{1+\epsilon'_i(1-\cos\boldsymbol{\Theta}')}$.

$$\frac{d\sigma_{KN}(\epsilon'_i,\boldsymbol{\Theta}')}{d\Omega'_f} = \frac{3}{16\,\pi}\frac{\sigma_T}{[1+\epsilon'_i(1-\cos\boldsymbol{\Theta}')^2]}\left[\epsilon'_i(1-\cos\boldsymbol{\Theta}') + \frac{1}{1+\epsilon'_i(1-\cos\boldsymbol{\Theta}')} + \cos^2\boldsymbol{\Theta}'\right].$$

Now only independent quantities appear.

- The cross section becomes smaller for increasing ϵ'_i.
- For $\boldsymbol{\Theta}' = 0$, it coincides with $\frac{d\sigma_T}{d\Omega_f}$ ($\epsilon'_f = \epsilon'_i$ for any ϵ'_i). However this corresponds to a vanishingly small number of interactions, as $d\Omega'_f \to 0$ for $\boldsymbol{\Theta}' \to 0$.
- Full backscatter is more probable for low energy photons.



Compton scattering- The Klein-Nishina cross section

Integrating over the solid angle, we obtain the total Klein-Nishina cross-section:

$$\sigma_{KN} = \frac{3}{4}\sigma_T \left\{ \frac{1+\epsilon_i}{\epsilon_i^3} \left[\frac{2\epsilon_i(1+\epsilon_i)}{1+2\epsilon_i} - \ln(1+2\epsilon_i) \right] + \frac{1}{2\epsilon_i}\ln(1+2\epsilon_i) - \frac{1+3\epsilon_i}{(1+2\epsilon_i)^2} \right\}$$

Asymptotic limits:

$$\sigma_{KN} \cong \sigma_T \left(1 - 2\epsilon_i + \frac{26\epsilon_i^2}{5} + \cdots \right); \qquad \epsilon_i \ll 1$$

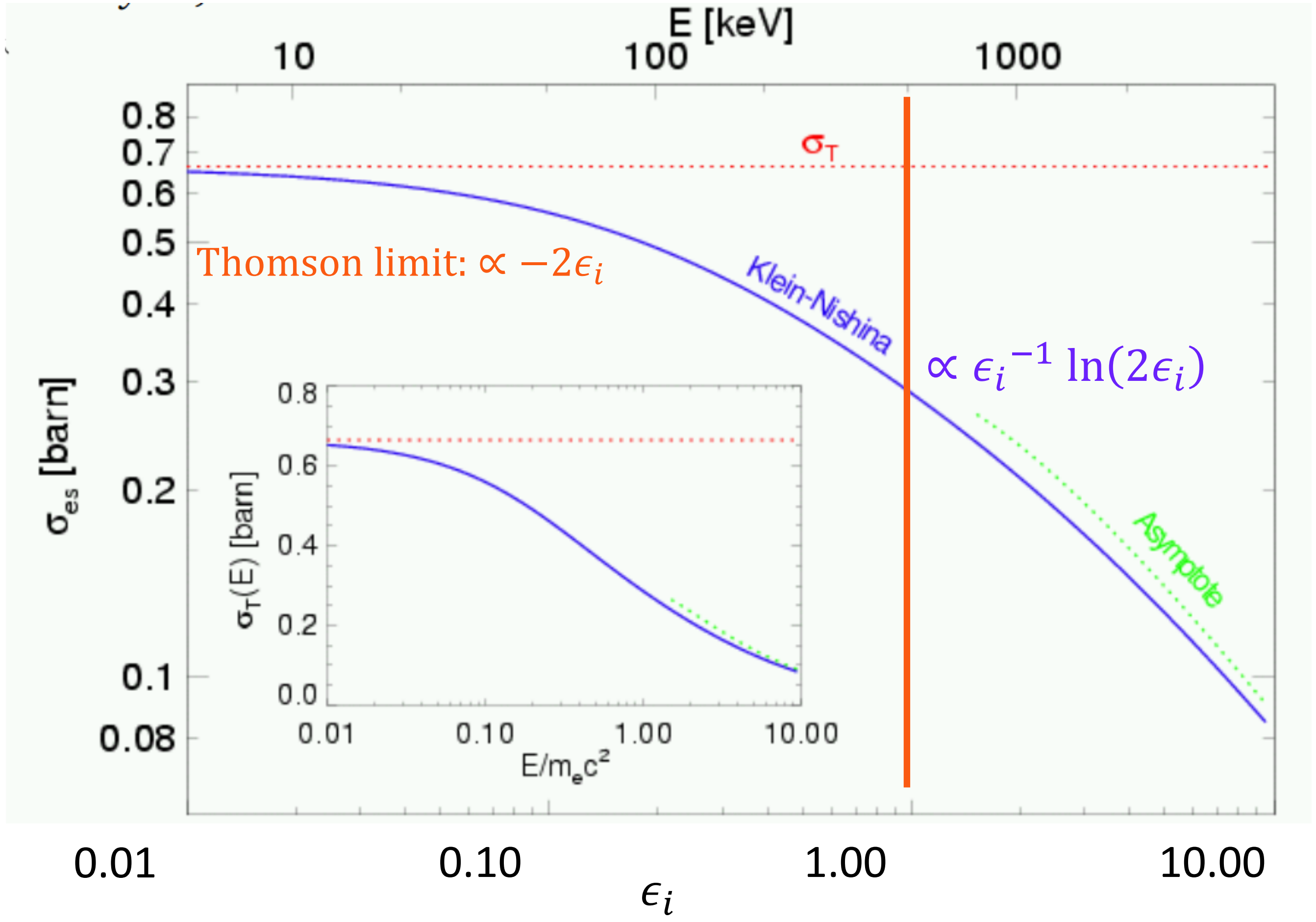
$$\sigma_{KN} \cong \frac{3}{8\epsilon_i}\sigma_T \left[\ln(2\epsilon_i) + \frac{1}{2} \right]; \qquad \epsilon_i \gg 1$$

We recognize a Thomson limit and a Klein- Nishina regime. Notice that, for scattering by non relativistic electrons,

$\epsilon_i = 1 \leftrightarrow h\nu_i = m_e c^2 = 0.511$ MeV

i.e. $\nu_i = \frac{0{,}511 \times 10^6 \text{ eV}}{4.1 \times 10^{-15} \text{ eV } s} \sim 10^{20}\text{Hz} = 10^8$ THz (Hard X/Gamma).

Klein Nishina cross section is smaller than σ_T , especially at high frequencies: scattering looses efficiency when photons are too energetic.



At which temperature is an electron relativistic?

Non relativistic electrons if $\frac{v^2}{c^2} \ll 1$.

In the *non-relativistic Maxwellian distribution*, the average kinetic energy is

$$\frac{1}{2} m_e v^2 = \frac{3}{2} kT_e \ , \qquad \text{thus } v^2 = 3kT_e/m_e \ .$$

We have $\beta^2 = \frac{v^2}{c^2} \ll 1$ for electron temperatures $T_e \ll \frac{m_e c^2}{3k} \sim 6 \times 10^9$ K, or

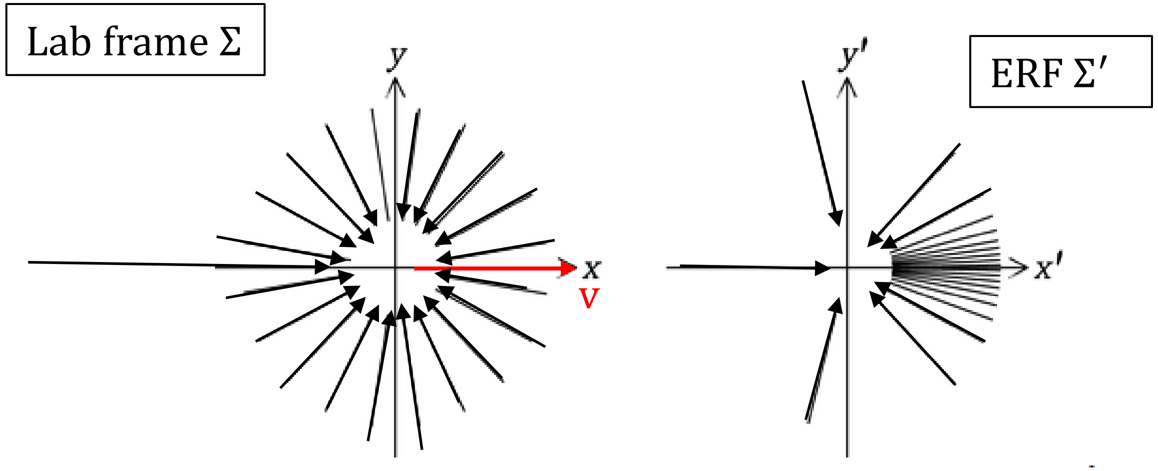
$kT_e \ll m_e c^2 = 0.511$ MeV ⇒ $T_e \ll 10^9$ K

Then $\gamma \sim 1$ and $\langle \gamma^2 \beta^2 \rangle \sim \left\langle 1 \cdot \frac{v^2}{c^2} \right\rangle = \frac{3kT_e}{m_e c^2} \ll 1.$

Relativistic electrons:

in a *relativistic thermal Maxwell–Jüttner distribution*,

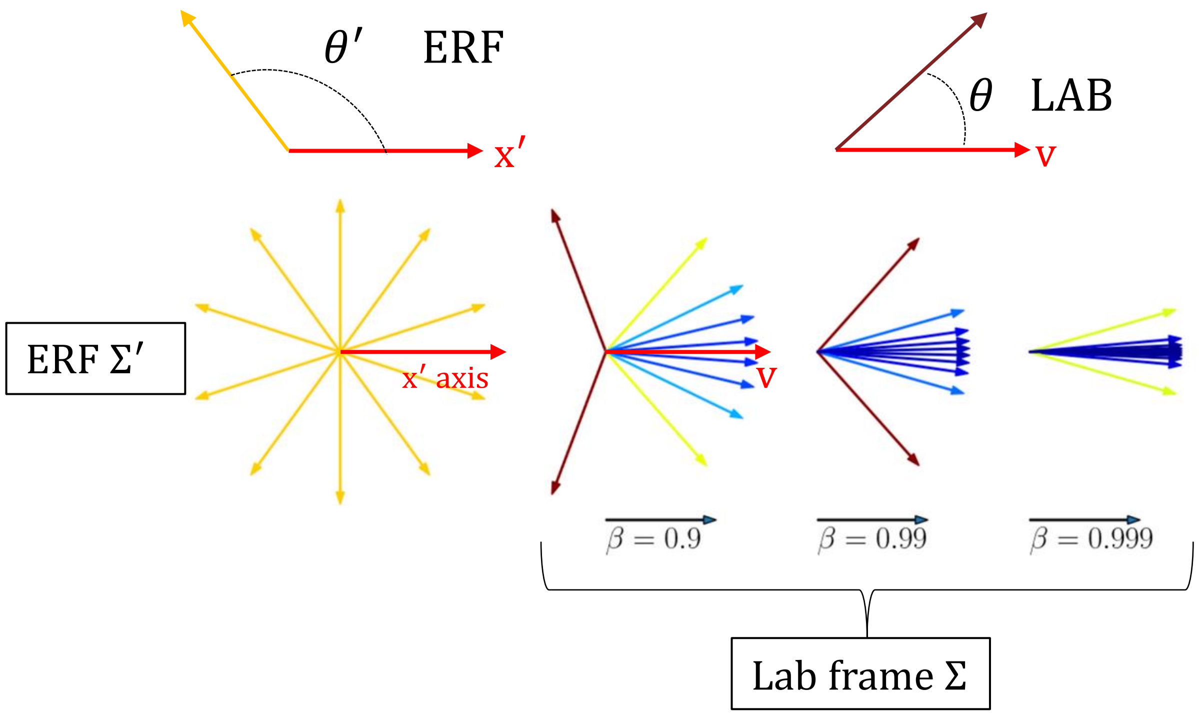
$$\langle \gamma^2 \beta^2 \rangle = 12 \left(\frac{kT_e}{m_e c^2} \right)^2 \ , \qquad T_e \gg 10^9 \text{ K}$$



$$\cos\theta' = \frac{\cos\theta - \beta}{1-\beta\cos\theta}\ ;$$

$$\sin\theta' = \frac{\sin\theta}{\gamma(1-\beta\cos\theta)}$$

- Even if the incident radiation is **isotropic in the LAB**, the relativistic electron sees *most* photons arriving from ahead (ERF), beamed in a cone of aperture $\sim 2/\gamma$.



$$\cos\theta = \frac{\cos\theta' + \beta}{1+\beta\cos\theta'}\ ;$$

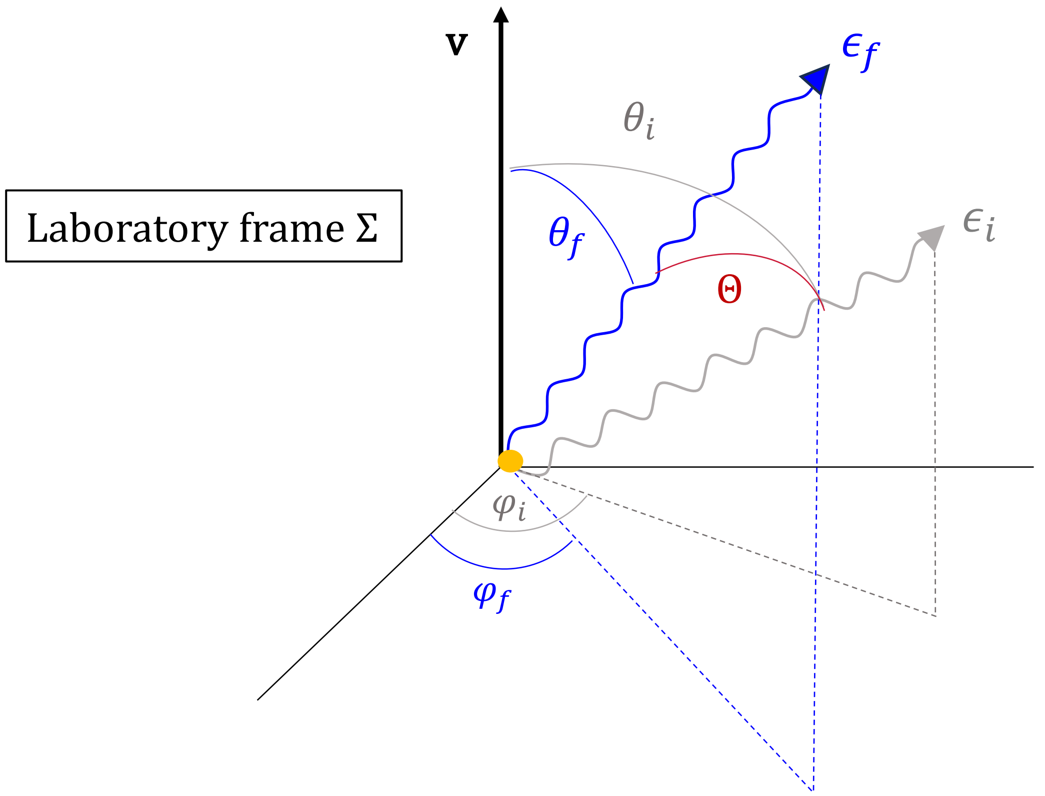
$$\sin\theta = \frac{\sin\theta'}{\gamma(1+\beta\cos\theta')}$$

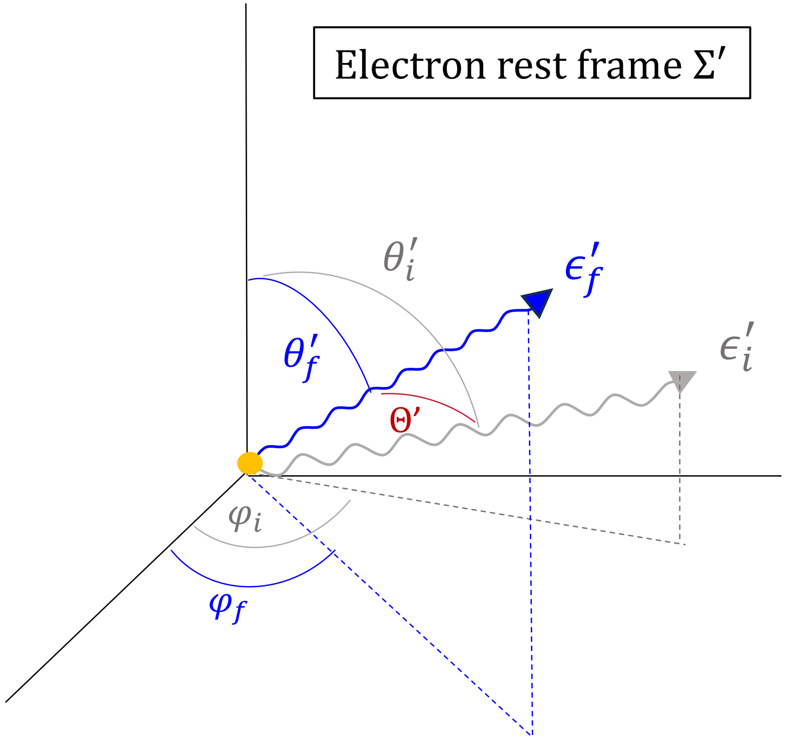
- The reverse is also true: even if the scattering (or the emission) by a relativistic electron is **isotropic in the ERF**, an observer in the LAB will see the scattered (or emitted) radiation as beamed in a cone of aperture $\sim 2/\gamma$.

Relativistic Compton scattering

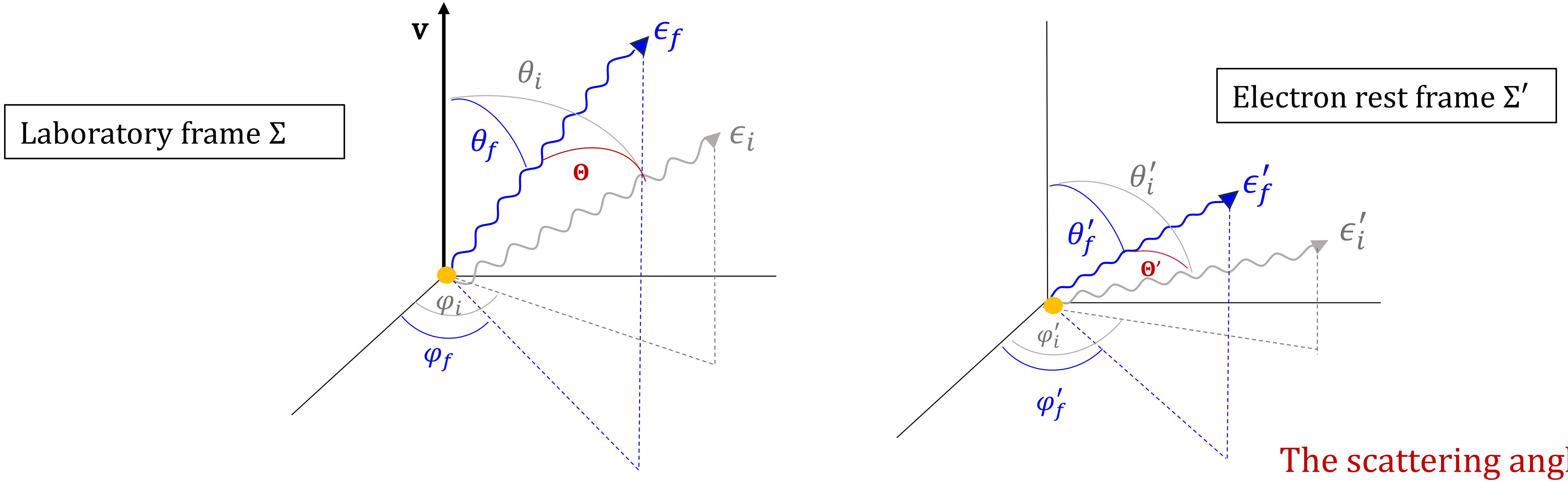
When the electron is **not at rest in the LAB**, but has an energy greater than the typical photon energy, there **can** be a transfer of energy **from the electron to the photon** . This process is called inverse Compton scattering.

We introduce the Electron Rest Frame Σ' , where the electron is initially at rest, and the Laboratory Frame Σ. Lorentz transformation are needed to describe how physical quantities change moving from one reference to the other.





Relativistic Compton scattering



The scattering angle is $\mathbf{\Theta}$ (or $\mathbf{\Theta}'$)

The photon **energy** transforms according to Doppler shift :

$\epsilon' = \gamma\epsilon(1 - \beta\cos\theta)$ θ= angle between electron velocity and photon direction.

$\epsilon = \gamma\epsilon'(1 + \beta\cos\theta')$

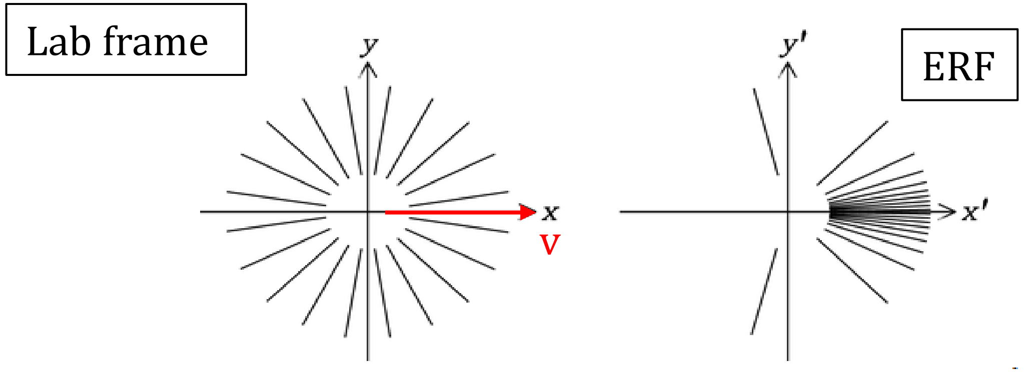
The Lorentz transformations dictate the **angle beaming**:

$\cos\theta' = \frac{\cos\theta - \beta}{1 - \beta\cos\theta}$; $\sin\theta' = \frac{\sin\theta}{\gamma(1-\beta\cos\theta)}$. The inverse formulae can be obtained inverting β with $-\beta$ and the angle θ with θ'.

Notice that for $\gamma \gg 1$, $\theta' \approx 0$ unless $\theta = 0$ (photons that, in the LAB, travel along the electron direction): in the ERF, a relativistic electron sees **most** photons arriving from ahead, beamed in a cone of aperture $\sim 2/\gamma$.

Relativistic Compton scattering

Photons that in the LAB frame come from every direction, in the ERF are beamed in a tiny cone of aperture $\sim 2/\gamma$.



The 4-momentum conservation equations in the ERF , Σ' are:

$$\begin{cases} \epsilon'_i + 1 = \epsilon'_f + \gamma & \text{Energy conservation} \\ \epsilon'_i \boldsymbol{n}'_i = \epsilon'_f \boldsymbol{n}'_f + \gamma \mathbf{v}/c & \text{Momentum conservation} \end{cases}$$

Taking the square of the second equation and using $\gamma^2\beta^2 = \gamma^2 - 1$, we obtain the relation between the photon energy in the ERF before and after the scattering with the electron:

$$\epsilon'_f = \frac{\epsilon'_i}{1+\epsilon'_i\,(1-\cos\boldsymbol{\Theta}')} \approx \epsilon'_i[1-\epsilon'_i\,(1-\cos\boldsymbol{\Theta}')] \approx \epsilon'_i \quad \text{for } \epsilon'_i \ll 1 \text{ (Thomson limit, leading to } \epsilon'_f \approx \epsilon'_i \text{)}$$

Where $\cos\boldsymbol{\Theta}' = \cos\theta'_i\cos\theta'_f + \sin\theta'_i\sin\theta'_f\sin(\varphi'_i - \varphi'_f)$

Relativistic Compton scattering

The Thomson limit in the ERF corresponds to the condition $\epsilon'_f = \epsilon'_i$. However, this *does not* imply that the electron is not exchanging energy with the photon in the LAB frame. Indeed, if we transform back to the LAB frame,

$$\epsilon_f = \gamma\epsilon'_f(1+\beta\cos\theta'_f) = \gamma\epsilon'_i(1+\beta\cos\theta'_f)[1-\epsilon'_i(1-\cos\boldsymbol{\Theta}')]$$

And using again the Lorentz transformation for the energies,

$$\epsilon_f = \gamma^2\epsilon_i(1-\beta\cos\theta_i)(1+\beta\cos\theta'_f)\,[1-\epsilon'_i(1-\cos\boldsymbol{\Theta}')]$$

Energy absorbed by the electron **recoil** in the ERF $= \Delta\epsilon'$ (Quantum mechanical effect)

$$\approx \gamma^2\epsilon_i(1-\beta\cos\theta_i)(1+\beta\cos\theta'_f)$$

in the Thomson limit $\epsilon'_i \ll 1$ (i.e., when in the ERF the photon energy is $h\nu'_i \ll m_e c^2$)

Relativistic boost effect

These equations show that , when in the LAB frame there is a collision between the photon and an ultrarelativistic electron ($\gamma \gg 1$), the **energy of the diffuse photon is much different that its initial energy**, even in the ERF Thomson limit.

Very useful formula: it can be shown that, properly averaging over the angles for an ISOTROPIC distribution of photons in the LAB frame, *in the Thomson limit ,*

$$\langle\epsilon_f - \epsilon_i\rangle \approx \frac{4}{3}\gamma^2\epsilon_i$$

or

$$\Delta\nu \approx \frac{4}{3}\gamma^2\nu_i$$

on average, an isotropic radiation is boosted to frequencies higher by a factor γ^2.

Extrema of the energy exchange in Thomson limit

- The maximum energy that can be gained is $\epsilon_f^{max} = \gamma^2 \epsilon_i (1+\beta)^2 = \boxed{\frac{1+\beta}{1-\beta}\epsilon_i}$, (*)

  for $\theta_i = \pi$, $\theta_f' = 0$: the photon is blue-shifted (face-on collision).

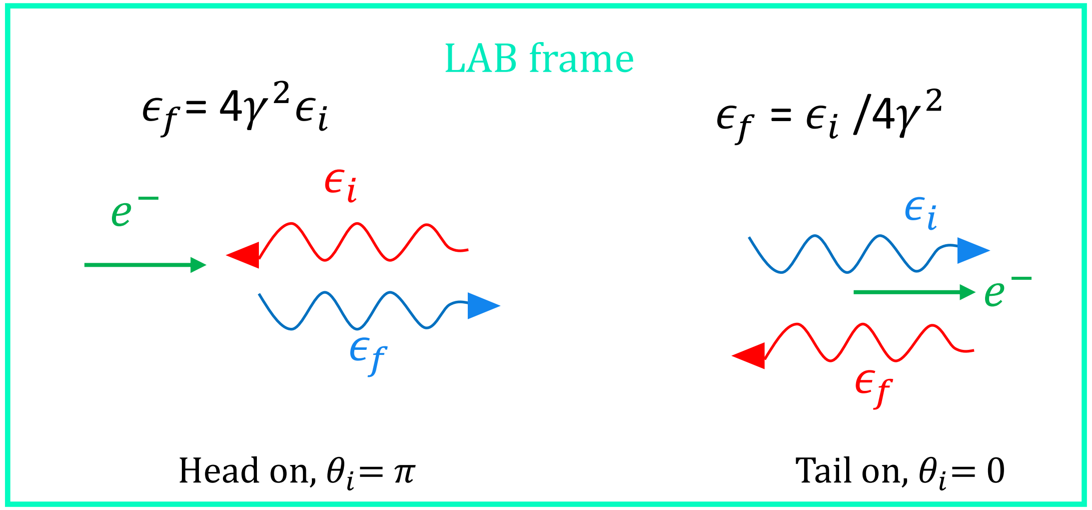
For $\gamma \gg 1$ $(\beta \sim 1)$, $\epsilon_f^{max} \cong 4\gamma^2 \epsilon_i$

(e.g., if $\gamma = 100$, a radio photon is blueshifted in the optical, $4\gamma^2 = 4 \times 10^4$)

- The minimum energy of the scattered photon is for $\theta_i = 0$, $\theta_f' = \pi$: the photon is redshifted (tail-on collision). In this case,

$$\epsilon_f^{min} = \gamma^2 \epsilon_i (1-\beta)^2 = \frac{1}{1-\beta^2}\epsilon_i (1-\beta)^2 = \frac{\epsilon_i (1-\beta)^2}{(1+\beta)(1-\beta)} = \boxed{\epsilon_i \frac{1-\beta}{1+\beta}} = \frac{\epsilon_i}{\gamma^2(1+\beta)^2} \to \frac{\epsilon_i}{4\gamma^2}$$

(for ultrarel. electrons, $(\beta \sim 1)$)



- The extrema of the energy variation in a single collision $\Delta\epsilon = (\epsilon_f - \epsilon_i)$ are:

$$\frac{-2|\beta|}{1+|\beta|} \leq \frac{\Delta\epsilon}{\epsilon_i} \leq \frac{2|\beta|}{1-|\beta|}$$

Notice: $\Delta\epsilon$ can be either Positive or negative.

However, the values inside this interval **do not have the same probability**. The general distribution of the scattered photon energies is the **Compton spectrum**. The final shape depends both on the properties of the incident radiation and on the velocity distribution of the electrons in the gas.

(*) We used $(1-\beta)(1+\beta) = \gamma^{-2}$

Relativistic Compton scattering – how to find the spectrum

Finding the relativistic Compton spectrum corresponds to determine how many photons there are, in the diffuse radiation, with energy ϵ_f , for each ϵ_f.

- To obtain the spectrum of the diffuse radiation, we should first know the spectrum of the incident radiation.

- Then, for the single photon ϵ_i scattered by a single electron with Lorentz factor γ , we know the LAB energy of the scattered photon as a function of its initial energy and of incident and scattering angle:

$$\epsilon_f = \gamma^2 \epsilon_i (1-\beta\cos\theta_i)(1+\beta\cos\theta'_f)\,[1-\epsilon'_i(1-\cos\boldsymbol{\Theta}')] \quad =$$

energy of a scattered photon wich collides with the electron with ϵ_i, θ_i *and is scattered as* ϵ_f, θ_f , $\boldsymbol{\Theta}$.

- The production of an ϵ_f (from an ϵ_i photon) from electron scattering has probability proportional to the (ERF) Klein Nishina differential cross section: it depends on the energy of the incident photon in the ERF (ϵ'_i) and on its initial/final directions in the ERF ($\boldsymbol{\Theta}'$).

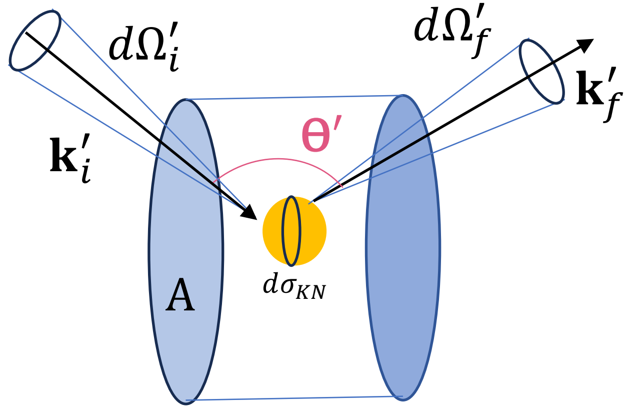
- We integrate over the distribution of initial photon energies ϵ_i and the directions θ_i of the incident radiation in the LAB, and over all the directions θ'_f of the diffuse radiation in the ERF, weighted with the Klein Nishina cross section **(*)**.

- Finally, we should integrate over the distribution of electron velocities. Usually the electrons are considered isotropically distributed.

(*) Applying the Klein-Nishina cross section, we are actually assuming that each photon undergoes a single scattering.

Scattering rate (single relativistic electron on radiation field)

$$\frac{dN(\mathbf{k}_i\rightarrow\mathbf{k}_f)}{dt} = \frac{1}{\gamma}\frac{dN(\mathbf{k}_i'\rightarrow\mathbf{k}_f')}{dt'} = \frac{1}{\gamma}\frac{dN(\mathbf{k}_i')\cdot Pb(\mathbf{k}_i'\rightarrow\mathbf{k}_f')}{dt'}$$



$$Pb(\mathbf{k}_i' \rightarrow \mathbf{k}_f') = \frac{number\ of\ targets \times effective\ target\ area(\Theta')}{Total\ area\ A} = \frac{1\times d\sigma_{KN}}{Total\ area\ A} = \frac{d\sigma_{KN}}{dV'\ /cdt'}$$

$$\frac{dN(\mathbf{k}_i\rightarrow\mathbf{k}_f)}{dt} = \frac{1}{\gamma}\frac{dN(\mathbf{k}_i')c\,d\sigma_{KN}}{dV'} = \frac{1}{\gamma}\underline{dn'(\mathbf{k}_i')}c\ d\sigma_{KN} = \frac{1}{\gamma}dn'(\mathbf{k}_i')c\ \frac{d\sigma_{KN}(\epsilon_i',\Theta')}{d\Omega_f'}\,d\Omega_f'$$

Number density

Consider a photon distribution in the LAB frame with number density $\mathrm{d}n(\mathbf{k}_i) = \frac{\mathrm{dN}(\mathbf{k}_i)}{\mathrm{dV}}$. Since dN and the 4-volume element dVdt are both Lorentz invariant, the number density $\mathrm{d}n(\mathbf{k}_i)$ must transform as a time (i.e., as an energy) when going from the LAB to the ERF. Using the Doppler formula for energy transformation, we have then:

$$\mathrm{d}n'(\mathbf{k}_i') = \mathrm{d}n(\mathbf{k}_i)\ \gamma(1-\beta\cos\theta_i)$$ (θ_i= angle between electron velocity and incident photon direction, in LAB frame)

$$\mathrm{d}n'(\epsilon_i',\Omega_i') = f'(\epsilon_i',\Omega_i')\,\mathrm{d}\epsilon_i'\,d\Omega_i' = \mathrm{d}n(\epsilon_i,\Omega_i)\,\gamma\,(1-\beta\cos\theta) = f(\epsilon_i,\Omega_i)\,\gamma\,(1-\beta\cos\theta_i)\mathrm{d}\epsilon_i\mathrm{d}\Omega_i$$

$$\frac{dN(\mathbf{k}_i\rightarrow\mathbf{k}_f)}{dt} = \frac{1}{\gamma}\boxed{f(\epsilon_i,\Omega_i)}\gamma(1-\beta\cos\theta_i)\,c\,\mathrm{d}\epsilon_i\mathrm{d}\Omega_i\frac{d\sigma_{KN}(\epsilon_i',\Theta')}{d\Omega_f'}\,d\Omega_f' \quad ; \quad \frac{dN(\epsilon_i\rightarrow\epsilon_f)}{\mathrm{d}\epsilon_i dt} = \int\frac{dN(\mathbf{k}_i\rightarrow\mathbf{k}_f)}{\mathrm{d}\Omega_i d\Omega_f' dt}\,\mathrm{d}\Omega_i d\Omega_f'$$

Incident radiation field

incident angles in the LAB
Diffusion angles in the ERF

The total **scattering rate is** $\frac{dN_s}{dt} = \int\frac{dN(\epsilon_i\rightarrow\epsilon_f)}{\mathrm{d}\epsilon_i dt}\,\mathrm{d}\epsilon_i$ (integral over the incident radiation field energies).

In the Thomson limit, $\boxed{\frac{dN_s}{dt} = c\sigma_T n}$ where n is the number density of photons, integrated over all ϵ_i (LAB frame).

Relativistic Compton spectrum

We found the scattering rate: $\frac{dN_s}{dt} = \int \frac{dN(\epsilon_i \to \epsilon_f)}{d\epsilon_i dt} d\epsilon_i = \int f(\epsilon_i, \Omega_i)\,(1-\beta cos\theta_i)\,c d\epsilon_i d\Omega_i \frac{d\sigma_{KN}(\epsilon_i', \Theta')}{d\Omega_f'} d\Omega_f'$ (*)

The power contained in the **scattered radiation** is , in its general form:

$$\frac{dE}{dt} = \int \frac{dN(\epsilon_i \to \epsilon_f)}{d\epsilon_i dt} \cdot \epsilon_f\,(\epsilon_i) d\epsilon_i$$

Energy of the scattered photons, weighted with the number of scatters per unit time that leads $\epsilon_i \to \epsilon_f$

Thomson limit (*neglect recoil in the ERF,* $h\nu_i' \ll m_e c^2$), $\epsilon_f\,(\epsilon_i) = \gamma^2 \epsilon_i (1-\beta\cos\theta_i)(1+\beta\cos\theta_f')$.

- Lorentz angle transformation: $\cos\theta' = \frac{\cos\theta - \beta}{1-\beta\cos\theta}$ $\Rightarrow 1+\beta cos\theta_f' = 1 + \beta \cdot \frac{(cos\theta_f - \beta)}{1-\beta cos\theta_f} =$

$= \frac{1-\beta cos\theta_f + \beta cos\theta_f - \beta^2}{1-\beta cos\theta_f} = \frac{\gamma^{-2}}{1-\beta cos\theta_f}$. In (*) we use σ_T, independent on ϵ_i' : $\int \frac{d\sigma_T}{d\Omega_f'} d\Omega_f' = \sigma_T$;

$$\Rightarrow \frac{dE}{dt} = \int \gamma^2 \epsilon_i f(\epsilon_i, \Omega_i) \cdot \frac{(1-\beta cos\theta_i)^2}{\gamma^2 (1-\beta cos\theta_f)} c\sigma_T\, d\epsilon_i d\Omega_i \,.$$

In the ERF, Thomson scattering has a backward-forward symmetry, thus $\langle\theta_f'\rangle = \frac{\pi}{2}$.

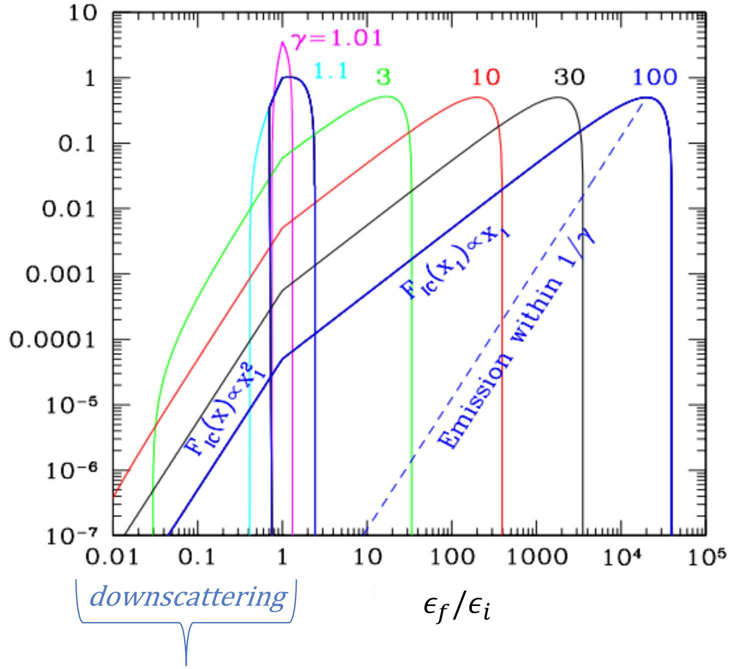
Then, $\langle cos\theta_f \rangle = \left\langle \frac{cos\theta_f' + \beta}{1+\beta\, cos\theta_f'} \right\rangle = \beta$, leading to $\langle(1-\beta cos\theta_f)\rangle = 1-\beta^2 = \gamma^{-2}$.

If the incoming photons are isotropically distributed, $\int (1-\beta cos\theta_i)^2\, d\Omega_i = 1 + \frac{\beta^2}{3}$.

$$\Rightarrow \quad \frac{dE}{dt} = \left(1+\frac{\beta^2}{3}\right)\gamma^2 c\sigma_T\, U_r$$

← power in the scattered radiation in Thomson limit.

with $U_r = \int \epsilon_i f(\epsilon_i)\, d\epsilon_i$ = radiation energy density before scattering, in the LAB.



IC spectrum by electrons of different γ on isotropic, monochromatic radiation of initial energy ϵ_i (from Ghisellini Fig. 5.8). Note: All spectra are normalized, just for comparison of shift in energy!

Energy exchange rate (single relativistic electron on radiation field)

In the LAB, the rate of energy **transferred** by the electron to the scattered photons is:

$$\frac{dE_s}{dt} = \int (\epsilon_f - \epsilon_i) \cdot \frac{dN(\mathbf{k}_i \to \mathbf{k}_f)}{dt\, d\Omega_i d\Omega'_f\, d\epsilon_i} d\Omega_i d\Omega'_f d\epsilon_i = \int (\epsilon_f - \epsilon_i) \cdot \frac{dN(\epsilon_i \to \epsilon_f)}{dt\, d\epsilon_i} d\epsilon_i$$

Energy variation in a single scattering

number of scatters per unit time that leads $\epsilon_i \to \epsilon_f$

The mathematical derivation is quite complex, and, for an isotropic photon distribution, brings to the result:

Energy exchange rate

$$\frac{dE_s}{dt} = c\, \sigma_T \left(\frac{4}{3} \gamma^2 \beta^2 - \gamma^3 \frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle} \right) U_r$$

This was obtained using the Klein-Nishina cross section, valid for the single scattering. It can be used only for low density plasma!

where $\langle \epsilon_i^2 \rangle$ is the average value of ϵ_i weighted over the isotropic initial photon distribution (LAB) (*) : $\langle \epsilon_i^2 \rangle = \frac{\int \epsilon_i^2 f_{iso} d\epsilon_i}{\int f_{iso} d\epsilon_i}$

Thomson limit and isotropic incident radiation: energy lost by the electron is the power of the scattered radiation minus the initial power of the radiation eventually scattered, everything averaged over all the angles:

$$\frac{dE_s}{dt} = \frac{dE}{dt} - c\sigma_T U_r = \left(1 + \frac{\beta^2}{3}\right) \gamma^2 c\sigma_T\, U_r - c\sigma_T U_r = c\sigma_T U_r \left[\gamma^2 \left(1 + \frac{\beta^2}{3}\right) - 1\right] = c\sigma_T U_r \left[\gamma^2 - 1 + \frac{\gamma^2\beta^2}{3}\right] = \frac{4}{3} c\sigma_T\, \gamma^2 \beta^2\, U_r > 0$$

(we used $\gamma^2 - 1 = \gamma^2\beta^2$)

Power in the scattered radiation

Initial power

Power always **provided by the relativistic electrons to the seed photons** [energy/time] in the Thomson limit: **INVERSE COMPTON**

(*) A photon distribution which is isotropic in the LAB, is not isotropic in the ERF of a relativistic electron.

Fractional photon energy change: Inverse Compton

Dividing the **exact** value of $\frac{dE_s}{dt}$ by the number of scatterings $\frac{dN_s}{dt}$, we obtain the average fraction of energy exchanged in the collisions **to the second order in ϵ_i** (neglecting terms of higher orders in ϵ_i)

$$\frac{\langle \epsilon_f - \epsilon_i \rangle}{\langle \epsilon_i \rangle} = \frac{4}{3}\gamma^2\beta^2 - \gamma^3 \frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle}$$

$\Rightarrow$ The energy released by the electron can be positive (**Inverse Compton**) or negative (direct Compton).

Relativistic **boost,**
Radiation heating

Quantum-mechanical **recoil** term,
always subtracting energy to the photons:
Radiation cooling

The sign depends on γ, but *also on the photon spectrum* through the ratio $\frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle}$. For any reasonable, isotropic distribution of photons, $\frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle} = \langle \epsilon_i \rangle$. These quantities are in the LAB. The fractional change is:

$$\frac{\langle \epsilon_f - \epsilon_i \rangle}{\langle \epsilon_i \rangle} \sim \gamma^2\beta^2 - \gamma^3 \langle \epsilon_i \rangle = \gamma^2 \left(\beta^2 - \gamma\langle \epsilon_i \rangle\right)$$

➡ opposition of relativistic boosting (photon heating) and quantum-mechanical electron recoil (photon cooling).

- **Non relativistic electrons**: for $\beta^2 \sim 0, \gamma \sim 1$, $(0 - \gamma\langle \epsilon_i \rangle) < 0$ **for any $\langle \epsilon_i \rangle$** (direct Compton: electrons recoil, photons cool down, **also** for $h\nu_i < m_e c^2$)
- **Ultrarelativistic electrons**: for $\beta^2 \sim 1$, $\left(1 - \gamma \frac{\langle h\nu_i \rangle}{m_e c^2}\right) < 0$ for $\gamma\langle h\nu_i \rangle > m_e c^2$: electrons recoil *only if* the incoming photons have energy larger than their rest energy.

Incident photon energy $h\nu_i'$ and electron energy as seen in the ERF!

Excursus on Thermal equilibrium in a diffusion-dominated cloud.

Our results outline an interesting thermodynamical property of the radiation contained in a diffusion-dominated cloud. Suppose the electrons in the cloud are in thermal equilibrium at T_e, but not in equilibrium with the radiation. Assume for simplicity that $kT_e \ll m_e c^2$ (non relativistic electrons).

From the non-relativistic Maxwellian distribution, the average kinetic energy is $3kT_e/2$ (thus $v^2 = 3kT_e/m_e$) : then $\gamma \sim 1$ and $\langle \gamma^2 \beta^2 \rangle = \left\langle \frac{v^2}{c^2} \right\rangle = \frac{3kT_e}{m_e c^2} \ll 1$

$\Rightarrow \frac{\langle \epsilon_f - \epsilon_i \rangle}{\langle \epsilon_i \rangle} = \frac{4}{3}\gamma^2\beta^2 - \gamma^3 \frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle} = \frac{4kT_e}{m_e c^2} - \frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle}$ $\Rightarrow$ transfer of energy from electrons to photons when $1 \gg \frac{4kT_e}{m_e c^2} > \frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle}$, and viceversa.

Photons blueshifted (IC) when their energy is $\langle \epsilon_i \rangle = h\nu_i/m_e c^2 \ll \frac{4kT_e}{m_e c^2}$, i.e. $h\nu_i \ll kT_e \ll m_e c^2$. Recoil negligible. The transfer ceases when

$\frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle} = \frac{4kT_e}{m_e c^2}$ which corresponds to the value of $\frac{\langle \epsilon_i^2 \rangle}{\langle \epsilon_i \rangle} = \langle \epsilon_i \rangle =$ u (energy density) for a Wien distribution with very negative chemical potential !

So the thermal equilibrium between electrons and photons is possible only if photons are forced to a Wien with $\mu \ll 0$. A similar results also hold for ultra relativistic electrons.

Similarities with the Synchrotron energy loss. Total cooling time.

For isotropic radiation and in the Thomson limit, note the similarity between the Compton and Synchrotron energy loss rates:

$P_{Compton} = \frac{4}{3} c\sigma_T \gamma^2\beta^2 \mathrm{U}_r$ $\qquad$ $P_{Synchrotron} = \frac{4}{3} c\sigma_T \gamma^2\beta^2 \mathrm{U}_B$ $\qquad$ (radiation and magnetic energy densities)

If the relativistic electrons are in a region with radiation and magnetic fields, they will emit **both syncrotron and Inverse Compton**. The ratio of the two luminosities are given by the ratio : $\frac{L_{syn}}{L_{IC}} = \boxed{\frac{P_{syn}}{P_{IC}} = \frac{U_B}{U_r}}$

This is true unless one of the two processes is inhibited, for example:

- At low frequencies (photon energies), synchrotron self-absorption compensates synchrotron emission;
- At high frequencies, electrons scatter in the Klein- Nishina regime, where the scattering is quite ineffective: the scattering decreases as the IC Compton emission does.

Their contribution sum up: energy loss = $\left(\frac{dE}{dt}\right) = \frac{4}{3} c\sigma_T \gamma^2\beta^2 (U_r + U_B)$.

The increased cooling rate implies that the electron radiative lifetime is reduced:

$$t_{lifetime} = \frac{E}{dE/dt} = \frac{3\gamma m_e c^2}{4c\,\sigma_T\,\gamma^2\beta^2\,(U_r+U_B)} \approx \frac{5.1\times10^5 eV}{2.99\times10^{10}\mathrm{cm\,s^{-1}} \times 6.65\times10^{-25}\mathrm{cm^2}\,\gamma(U_r+U_B)} = \frac{2.56\times10^{19}\ eV\ \ \times3\times10^{-8}\ yr}{\gamma(U_r+U_B)[eV\,cm^{-3}\,]cm^3} = \frac{7.6\times10^{11}\ eV\ yr}{\gamma(U_r+U_B)[eV\,cm^{-3}\,]cm^3}$$

(we used $\gamma^2\beta^2 = \gamma^2 - 1 \sim\gamma^2$ for relativistic electrons) [Typical values of U_r in Milky Way are 10^{-3} $-$ 10 eV cm^{-3}]

Synchrotron-Self Compton (SSC)

Synchrotron self-Compton radiation results from inverse-Compton scattering of synchrotron radiation by the same relativistic electrons that produced the synchrotron radiation. : the relativistic electrons that are the source of low energy photons (Synchrotron emission) are also responsible for Compton scattering these photons to X- and γ-ray energies

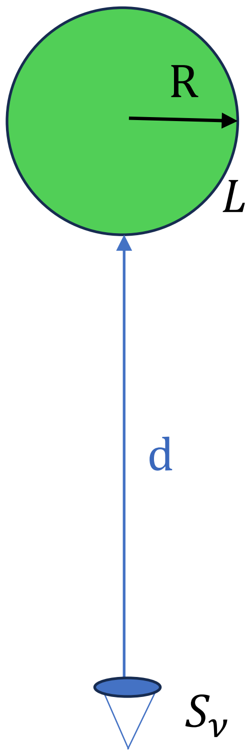
In a synchrotron emitting source, the magnetic energy density ($B^2/8\pi$) must be greater than the photon energy density for synchrotron losses to dominate over inverse Compton losses.

The requirement for synchrotron losses to dominate (for a spherical, stationary source of luminosity L, radius R, and distance d) is

$$U_B = \frac{B^2}{8\pi} > \frac{L}{c4\pi R^2} = \frac{4\pi d^2 \int_{\nu_{min}}^{\nu_{max}} S_\nu d\nu}{c\, 4\pi R^2} = U_{rad}$$

(remember $U_{rad} = \frac{\text{Intensity}}{\text{c}}$)

Spherical cloud with radius R filled with magnetized relativistic plasma, located at a distance d from the observer. The low-energy synchrotron photons may be up-scattered by the relativistic electrons via IC:



where the observed flux density for the optically thin part of the synchrotron spectrum is a power law, $S_\nu \propto \nu^{-\alpha}$.

Assume $s < 1$, $\nu_{max} \gg \nu_{min}$ and $\nu_{min} \sim \nu_a$, where ν_a is the synchrotron self-absorption frequency. Under these assumptions,

$$\int_{\nu_{min}}^{\nu_{max}} S_\nu d\nu \approx \frac{\nu_{max} S(\nu_{max})}{1-\alpha} = \frac{\nu_a S(\nu_a)}{1-\alpha} \left(\frac{\nu_{max}}{\nu_a}\right)^{1-\alpha} \quad ;$$

we define $x = \frac{1}{1-\alpha}\left(\frac{\nu_{max}}{\nu_a}\right)^{1-\alpha}$ where x is typically of order 10 .

The requirement can be written $\frac{B^2}{8\pi} > \frac{\nu_a}{c} S(\nu_a)\, x \frac{d^2}{R^2}$

Synchrotron-Self Compton (SSC)

At the self-absorption frequency, the **specific flux** emerging from the source is approximately πB_{ν_a}, where B_ν is the Planck function.

In an optically thick medium we can use the R-J approximation of the Planck's function, $B_\nu = 2kT_B \frac{\nu^2}{c^2}$

Therefore, the observed flux density is $S(\nu_a) \approx \pi B_{\nu_a} \frac{R^2}{d^2} = 2\pi k T_B \frac{\nu_a^2}{c^2} \frac{R^2}{d^2}$ where $kT_B = 2m_e c^2 \left(\frac{\pi \nu m_e c}{3eB}\right)^{1/2}$ (**)(see lecture on Synchrotron self absorption)

The requirement for synchrotron to dominate over Compton becomes: $\frac{B^2}{8\pi} > \frac{2\pi\nu_a^3}{c^3} kT_B x$;

combining this with (**) (calculated at $\nu = \nu_a$), we eliminate the magnetic field strength. Synchrotron emission dominates over IC if

$$(kT_B)^5 < \frac{c^{13} m_e^6}{9e^2 \nu_a x}$$

The ratio of inverse Compton to synchrotron cooling, $\frac{L_{IC}}{L_{syn}}$, is proportional to the quantity $(x\nu_a T_B^5)$: it is very sensitive to the brightness temperature and not much else. For typical values of x and ν_a, the brightness temperature limit to avoid the SSC Compton catastrophe is

$$\Rightarrow \quad T_B < 1.5 \times 10^{12} \left(\frac{\nu_a}{10^9\,\mathrm{Hz}}\right)^{-1/5} \left(\frac{x}{10}\right)^{-1/5} \mathrm{K}$$

Then there is an upper limit of $\approx 10^{12}$ K on the brightness temperature of a radio source, typically the compact core of a quasar, for synchrotron losses to dominate over IC. The "Compton catastrophe" refers to the rapid Compton cooling that would occur if the brightness temperature exceeds this limit.

The SSC Catastrophe happens in bright and compact radio sources where relativistic beaming plays a role and enhances the energy density of the radiation field.(e.g. in Blazar sources)

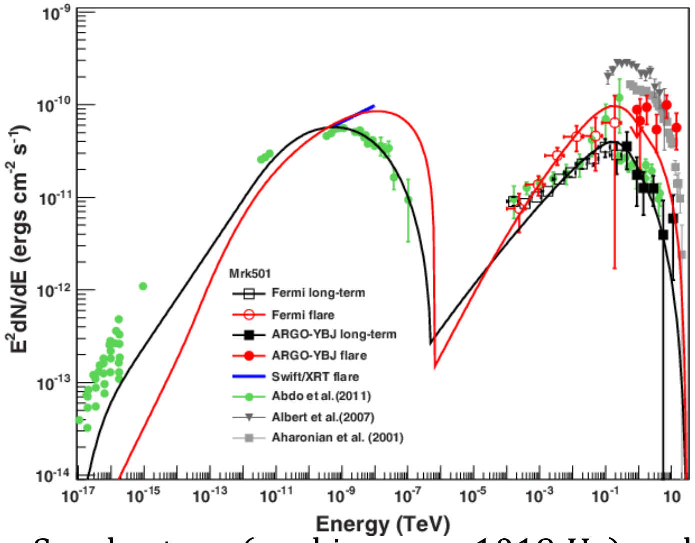
References : The fact that quasar brightness temperatures do not exceed these sorts of values led Kellerman & Pauliny-Toth, Ap. J., 155, L71 (1969) to propose this mechanism as the explanation. For a contrary point of view, arguing that the Compton catastrophe is not an important mechanism limiting the brightness temperature, see Readhead, Ap. J., 426, 51 (1994)

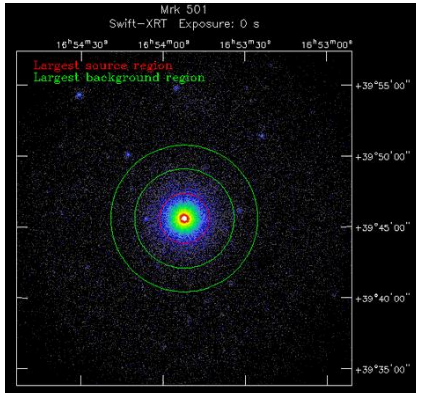
Synchrotron-Self Compton (SSC)

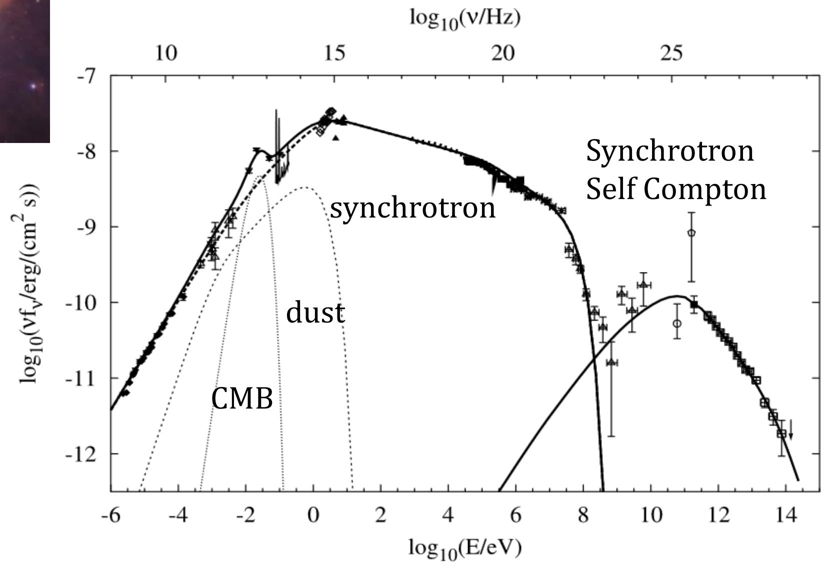
Independent on electrons number

Scattering relevant for high density of electrons and photons. The relation $\frac{U_{rad}\,P_{syn}}{P_{IC}} = U_B$ implies that multiplying the density of relativistic electrons by some factor f multiplies both the synchrotron power P_{syn} and its contribution to U_{rad} by f, so the SSC power P_{IC} is $\propto f^2$.

The SSC radiation also contributes to U_{rad} and leads to significant second-order scattering as the SSC contribution to U_{rad} approaches the synchrotron contribution in compact sources. This runaway positive feedback is a very sensitive function of the source brightness temperature: IC losses very strongly cool the relativistic electrons if the source brightness temperature exceeds $T_B \sim 10^{12}$ K in the rest frame of the source.







Synchrotron (peaking near 1019 Hz) and SSC (peaking near 1027 Hz) spectra of Mrk 501 Thin curve : the best-fit SSC model. Thick points X-ray data and the γ-ray data are plotted as points with error bars.
The relative heights of the two peaks indicate their relative contributions to U_{rad}.

Crab nebula emission from radio up to TeV Gamma rays.

Synchrotron-Self Compton (SSC)

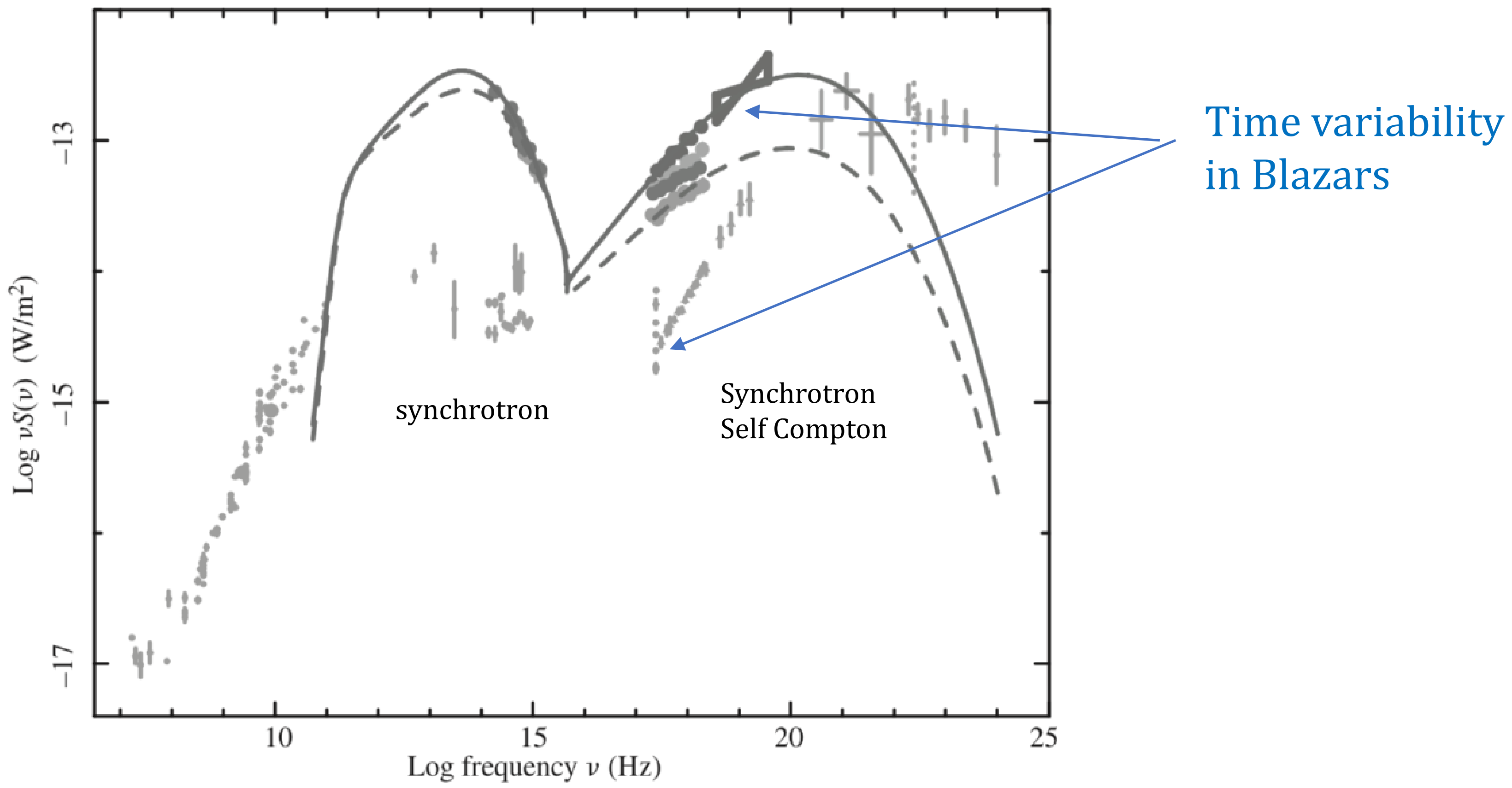


Fig. 9.6: Spectral energy distribution (SED) of blazar 3C454.3 over many wave bands from many observatories. The two-peaked character of Fig. 9.5 is clearly evident in the data and in the simple one-zone model fits shown as solid and dashed lines. The filled circles signify recent quasi-simultaneous subsets of radio, optical, x-ray, and gamma-ray data, the latter three being mostly from the Swift gamma-ray burst observatory. The smaller points represent nonsimultaneous observations in the literature. The data show large intensity variations across most frequency bands. [P. Giommi *et al.*, *A&A*, **456**, 911 (2006)]

Comptonization

A population of photons that encounters a region containing relativistic free electrons will find its spectrum modified as a result of inverse Compton scattering, given sufficient optical depth.

If the electrons are on average more (less) energetic than photons, then the photons will on average be up-scattered (down-scattered) to higher (lower) energies.

If the evolution of the spectrum of the source is dominated by Compton scattering, the process is referred to as Comptonization.

Examples of sources in which such conditions are found include the **hot gas in the vicinity of binary X-ray sources**, the **hot plasmas in the nuclei of active galaxies**, the hot **intergalactic gas in clusters of galaxies** and the early evolution of the hot primordial plasma.

In a cloud of isotropic electrons, the scattering rate we obtained is still valid.

Key points:

Effective in rarefied plasma (no significant competing cooling mechanism)

Hot electrons (must transfer energy to low energy photons)

Total number of photons is conserved, i.e. possible BB photon distribution is modified

Equilibrium is achieved in "saturated sources" (i.e. when, after a number of scatterings, the energy of the radiation is balanced with that of the electrons: $h\nu = 4KT$)

Thermal Comptonization

We have seen that for a *non-relativistic electron distribution*, the energy exchange photon/electron in a single scattering is

$$\frac{\Delta\mathcal{E}}{\mathcal{E}} \equiv \frac{\langle\epsilon_f - \epsilon_i\rangle}{\langle\epsilon_i\rangle} = \frac{4}{3}\gamma^2\beta^2 - \gamma^3\frac{\langle\epsilon_i^2\rangle}{\langle\epsilon_i\rangle} = \frac{4kT_e}{m_e c^2} - \frac{\langle\epsilon_i^2\rangle}{\langle\epsilon_i\rangle} \sim \frac{4kT_e}{m_e c^2} - \langle\epsilon_i\rangle \quad (**) .$$

Suppose that $\langle\epsilon_i\rangle \ll \frac{4kT_e}{m_e c^2}$, the fraction of energy gained per scattering is:

$\frac{\Delta\mathcal{E}}{\mathcal{E}} = \frac{4kT_e}{m_e c^2} \Rightarrow \Delta\mathcal{E} = \mathcal{E}\frac{4kT_e}{m_e c^2}$. The degree to which multiple Compton scattering change the initial photon spectrum is given by a parameter:

y= (*average fractional energy exchange per scattering*) × (*mean number of scatterings*)$= \frac{\Delta\mathcal{E}}{\mathcal{E}} \times \max(\tau_s, \tau_s^2)$

The mean number of scattering N is τ_s^2 for $\tau_s \gg 1$, and τ_s otherwise, from the random walk (*).

So $y = \frac{4kT_e}{m_e c^2}\max(\tau_s, \tau_s^2)$. After N scatterings, the photon energy will be $\mathcal{E}(N) = \mathcal{E}_i e^{N\left(\frac{4kT_e}{m_e c^2}\right)} = \mathcal{E}_i e^y$.

- **Saturation: high optical depth.**

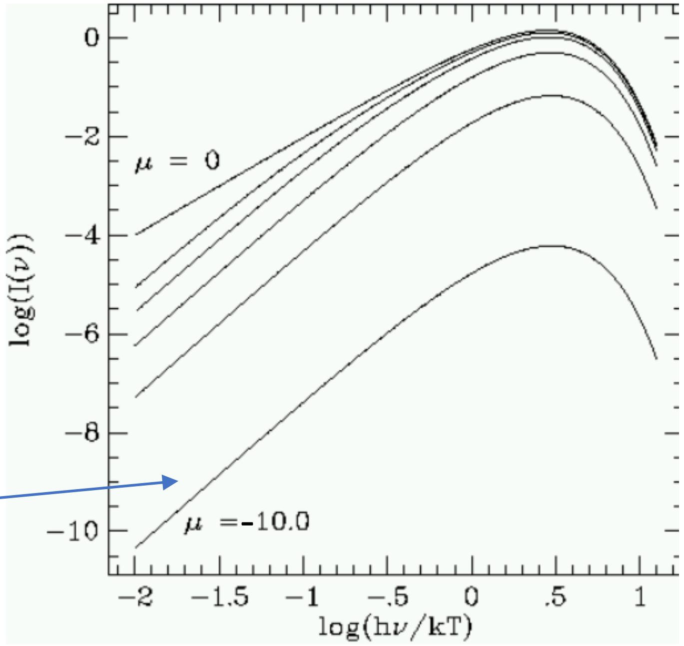
When $\mathcal{E}(N) = \frac{4kT_e}{m_e c^2}$, from (**), the photon stops gaining energy, and Comptonization is said to be saturated.

This thermal equilibrium occours at a critical value of y corresponding to a critical value of $\tau_s \gg 1$.

$$\frac{4kT_e}{m_e c^2}\cdot\frac{1}{\mathcal{E}_i} = e^{y_{crit}} \quad \text{or} \quad \tau_{crit} = \sqrt{\frac{m_e c^2}{4kT_e}\ln\left(\frac{4kT}{\mathcal{E}_i}\right)} \quad .$$

The photons obey a Bose-Einstein distribution : $u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(\frac{h\nu}{kT} - \mu\right) - 1}d\nu$. $(\mu < 0)$

For $\left(\frac{h\nu}{kT} - \mu\right) \gg 1$, $u(\nu)d\nu \sim \frac{8\pi h\nu^3}{c^3}e^{-\frac{h\nu}{kT}}e^{\mu}$, i.e. the Wien's law modified by e^{μ}



(*) Rybicki Lightman page 36

Repeated scatterings at low optical depth.

Photon's energy is multiplied by $\frac{4}{3}\gamma^2$ on average upon each scattering from an electron of Lorentz factor γ (remember the useful formula of the average, $\langle\nu_f\rangle \approx \frac{4}{3}\gamma^2\nu_i$).

After k scatterings, $\nu_f = \nu_i\left(\frac{4}{3}\gamma^2\right)^k$ and, whatever the width $\Delta\nu_i$ of the seed spectrum is, the scattered spectrum will be **increased in width** by the same factor: $\Delta\nu_f = \nu_i\left(\frac{4}{3}\gamma^2\right)^k$.

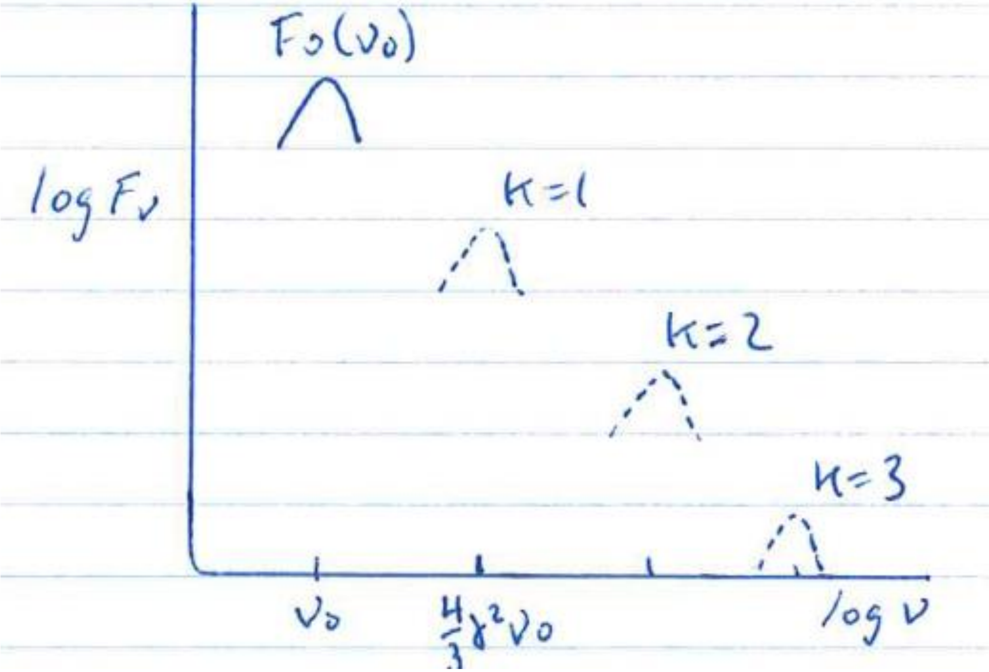
The probability that a photon is scattered k times is τ_s^k if τ_s is small. Therefore, the scattered spectrum will have a specific flux that can be approximated as $F(\nu) = F_i(\nu_i)\,\tau_s^k$

$$\ln\left[\frac{F(\nu)}{F_i(\nu_i)}\right] = k\ln\tau_s$$

$$\ln\left(\frac{\nu}{\nu_i}\right) = k\ln\left(\frac{4}{3}\gamma^2\right) \Rightarrow \textit{write } k \textit{ as a fuction of } \nu$$

$$\ln\left[\frac{F(\nu)}{F_i(\nu_i)}\right] = \frac{\ln\left(\frac{\nu}{\nu_i}\right)}{\ln\left(\frac{4}{3}\gamma^2\right)}\cdot\ln\tau_s$$

$$F(\nu) = F_i(\nu_i)\left(\frac{\nu}{\nu_i}\right)^{-\alpha} \quad \text{where } \alpha \equiv -\frac{\ln\tau_s}{\ln\left(\frac{4}{3}\gamma^2\right)}$$



If the seed spectrum is distributed over a wide range $\Delta\nu_i$ the scattered spectrum at frequency ν can be generalized by integrating over $F_i(\nu_i)$ for $\nu_i < \nu$. Only seed photons with $\nu_i < \nu$ contribute to $F(\nu)$.

$$\nu\,F(\nu) = \int_0^\nu F_i(\nu_i)\left(\frac{\nu}{\nu_i}\right)^{-\alpha} d\nu_i \qquad \text{or } F(\nu) = \nu^{-(\alpha+1)}\int_0^\nu F_i(\nu_i)\,\nu_i^\alpha\,d\nu_i$$

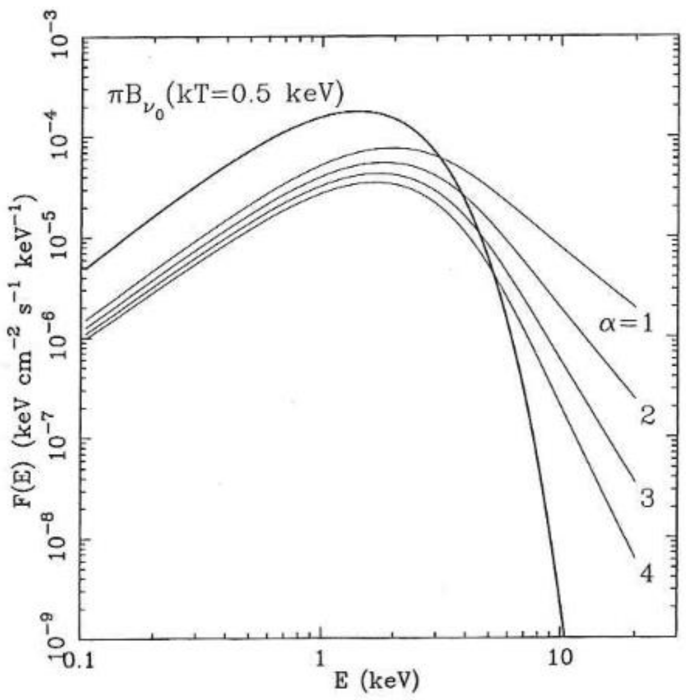
An example is the Comptonized Blackbody, $F(\nu)=\pi\nu^{-(\alpha+1)}\int_0^{\nu} B_{\nu_i}(T)\,\nu_i^{\alpha}\,d\nu_i$.

It has a power law tail of index $\alpha + 1$.

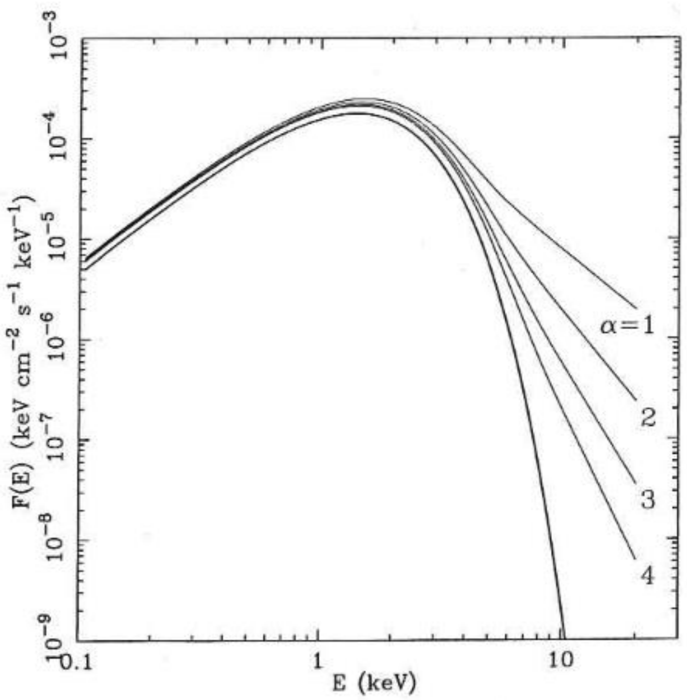
In this approximation it is required that $\tau_s \ll 1$ and $h\nu \ll \gamma m_e c^2$. From the definition of α, it follows that $\left(\frac{4}{3}\gamma^2\right)^{\alpha}\tau_s = 1 \Rightarrow \left(\frac{4}{3}\gamma^2\right)\tau_s = \left(\frac{4}{3}\gamma^2\right)^{1-\alpha}$.

This expression is less than 1 as long as $\alpha > 1$. The left side is exactly the Compton y parameter for the case $\tau_s < 1$.

Thus, large α correspond to small y.



Blackbody and Compton scattered spectra for different α.



Sum of Blackbody and Compton scattered spectra.

$T_{rad} \sim 6 \times 10^6 K$

The exact description of the photon scattering (spectrum originated by different values of the y-parameter) is obtained in the phase-space through the Kompaneet's equation. In the phase space, the «cloud» formed by the representative points of the real space is similar to a diffusion process. If the electrons are in non-reklativistic thermal equilibrium, the phase space temporal evolution of the occupation number $n(\nu) = \frac{u(\nu)c^3}{8\pi h\nu^3}$,

and defining $x = \frac{h\nu}{kT}$

$$\frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4\left(n + n^2 + \frac{\partial n}{\partial x}\right)\right]$$

Recoil effect

stimulated emission (*)

Doppler motion

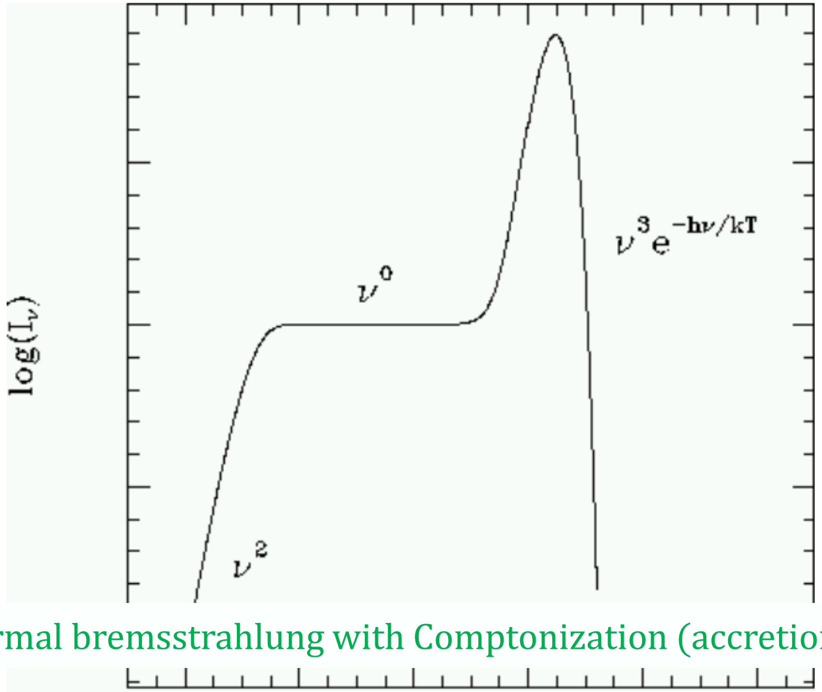
(*) *If photon occupation numbers are not small, consider that photons are spin 1 bosons, and bosons like to clump: stimulated transitions ensure that a final state is more probable if it is already occupied!*

In the limit of many scatterings, we obtain the equilibrium Bose-Einstein distribution with non-zero chemical potential (see saturation).

Emission spectra come out as power-law with index $3 + m$ where $m = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{1}{y}}$

$+$ for $y \gg 1$, $-$for $y \ll 1$.

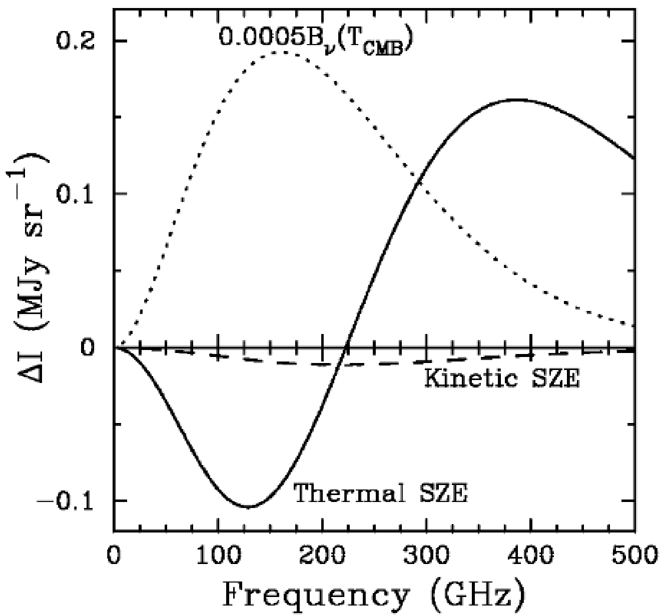
In the limit of many scatterings, $3 + m \sim 3$.



Thermal bremsstrahlung with Comptonization (accretion disks)

Example. Thermal Sunyaev Zeldovich effect

CMB blackbody spectrum is distorted by the presence of galaxy clusters, the largest gravitationally-collapsed (virialized) structures in the universe. Gas falling into the gravitational potential well of these clusters is heated to roughly $10^8\ K$, and becomes **ionized**. CMB photons pass through this plasma, and as many as 1-2% of them can be IC scattered by the hot gas. On average the energy of the scattered photons is **increased**, spectrally distorting the CMB in a characteristic way, known as the thermal SZ effect.



In the sparse astrophysical plasma, of ICM, interactions between particles can be considered almost absent => no photon absorption or creation (photon conservation), but only a redistribution of their energies.

Galaxy clusters contain hot atmospheres with $T_e \sim$ 6 KeV$\sim 7 \times 10^8\ K$, $n_e \sim 10^{-3} cm^{-3}$, size L~ 1 Mpc. ⇒ Non-relativistic electrons (Maxwell Boltzmann distribution) . Decrement In Rayleigh-Jeans region, increment in Wien region. Solve the Kompaneets equation with some **simplification**:

1) Scattering chance is small, only 1% per CMB photon: optical depth τ low for hot ICM e$^-$ gas. **Photons scatter once.** The central intracluster optical depth is indeed $\tau_e \approx n_e \sigma_TL\sim 10^{-2}$.

2) Electron gas much hotter than CMB photons: the term $n + n^2$ negligible (no recoil, energy only transferred from electrons to photons⇒ $IC\ scattering$). Crossover frequency ~ **217 GHz** (x=3.83). ΔI/I$\sim 10^{-4}$